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Ministry of Education  
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of Book Affairs

# MATHEMATICS



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غير مصرح بتداول هذا الكتاب خارج  
وزارة التربية والتعليم الفني

**For Preparatory Year Two**

**First Term**

**Student's Book**

## **Authors**

**Mr. Omar Fouad Gaballa**

**Prof.Dr. Afaf Abo-ElFoutoh Salah      Dr. Essam Wasfi Roupheail**

**Mr. Mahmoud Yasser Al-Khatieb      Mr. Serafiem Elias Iskander**

**Revised by Samir Mohamed Sedawy**

**Counsellor of Mathematics**

**M. O. E**

**Mr. Gamal El shahed**



## بسم الله الرحمن الرحيم

### Dear students:

It is extremely great pleasure to introduce the mathematics book for second preparatory. We have been specially cautious to make your learning to the mathematics enjoyable and useful since it has many practical applications in real life as well as in the other subjects. This gives you a chance to be aware of the importance of learning mathematics, to determine its value and to appreciate the mathematicians roles.

This book sheds new lights on the activities as a basic objective. Additionally, we have tried to introduce the subject simply and excitingly to help attaining mathematical knowledge as well as gaining the patterns of positive thinking which pave your way to creativity .

This book has been divided into units, each unit contains lessons. Colors and pictures are effectively used to illustrate some mathematical concepts and the properties of figures. Lingual level of previous study has been taken into consideration .

Our great interest here is to help you get the information by your self in order to develop your self-study skills.

Calculators and computer sets are used when there's a need for. Exercises, practices, general exams, portfolios, unit test, general tests, and final term tests attached with model answers have been involved to help you review the curriculum completely.

Eventually, we hope getting the right track for the benefits of our students as well as for our dearest Egypt hoping bright future to our dearest students.

**Authors**



# Contents

## Unit One : Real Numbers

Revision :	2
Lesson One : The cube root of a rational number .....	5
Lesson Two : The set of irrational number $Q'$ .....	9
Lesson Three : Finding the approximate value of an irrational number.....	11
Lesson Four : The set of real numbers $R$ .....	16
Lesson Five : Ordering numbers at $R$ .....	19
Lesson Six : Intervals .....	21
Lesson Seven : Operations on the real numbers.....	27
Lesson Eight : Operations on the square roots.....	33
Lesson Nine : Operations on the cube roots.....	38
Lesson Ten : Applications on the real numbers.....	40
Lesson eleven : Solving equations and inequalities of first degree in one variable in $R$ .....	46
General Exercises .....	49
Activity .....	51
Unit Test .....	52

## Unit Two : Relation between two Variables

Lesson One : Linear relation of two variables .....	54
Lesson Two : The Slope of a line and real - Life applications .....	58
Unit Test    Activity .....	64

## Unit Three : Statistics

Lesson One : Collecting and organizing data .....	66
Lesson Two : The Ascending and descending cumulative frequency table and their graphical representation.....	70
Lesson three : Arithmetic Mean, Median and Mode .....	75
General Exercises .....	82
Activity .....	83
Unit Test .....	84

## Unit Four: Geometry

Lesson One : The medians of a triangle	86
Lesson Two: The Isosceles Triangle	92
Lesson Three : The Isosceles Triangle theorems	94
Lesson Four : Corollaries of isosceles triangle theorems	106
General Exercises	112
Activity	114
Unit Test	115

## Unit Five : Inequality

Lesson One : Inequality	117
Lesson Two: Comparing the measures of the angles of a triangle	121
Lesson Three : Comparing the lengths of sides in a triangle	126
Lesson Four : Triangle Inequality	132
General Exercises	134
Activity	136
Unit Test	137
Model Answers	138
General Exercises and General Tests	139



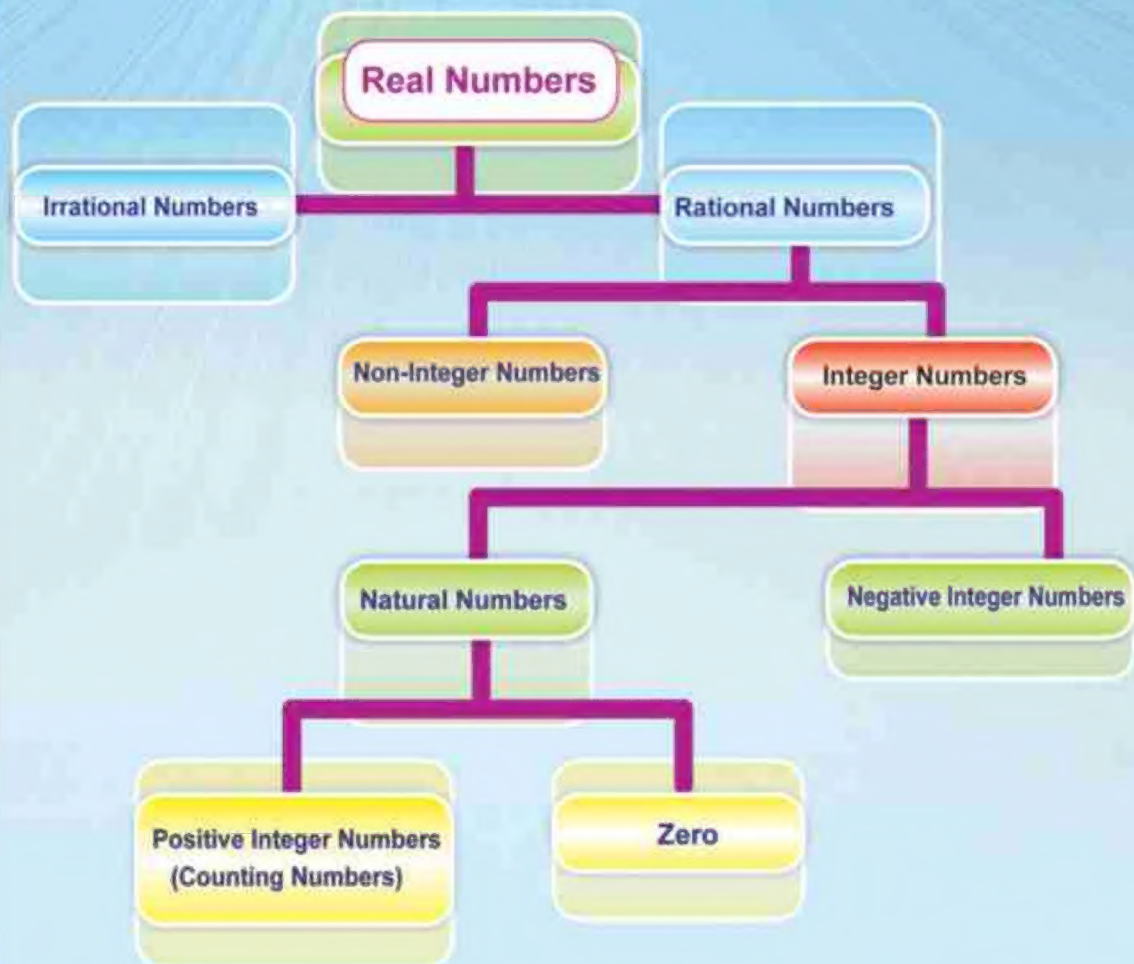
## The used Mathematical Symbols

<b>N</b>	The set of natural numbers	$\perp$	perpendicular to
<b>Z</b>	The set of integer numbers	$//$	parallel to
<b>Q</b>	The set of rational numbers	$\overline{AB}$	Line segment <b>AB</b>
<b>Q'</b>	The set of irrational numbers	$\overrightarrow{AB}$	Ray <b>AB</b>
<b>R</b>	The set of real numbers	$\leftrightarrow$ $\overleftrightarrow{AB}$	straight line <b>AB</b>
$\sqrt{a}$	Square root of number a	$m(\angle L)$	measure of angle L
$\sqrt[3]{a}$	Cube root of number a	$\sim$	Similarity
<b>[a , b]</b>	Closed interval	$<$	less than
<b>]a , b[</b>	Open interval	$\leq$	less than or equal to
<b>[a , b[</b>	Half-open (closed) interval	$>$	greater than
<b>]a , b]</b>	Half-open (closed) interval	$\geq$	greater than or equal to
<b><math>]-\infty, a]</math> <math>[a , \infty[</math></b>	Infinite interval	<b>P(E)</b>	probability of occurring event (E)
$\equiv$	is congruent to		

## UNIT ONE

# 1

# Real Numbers





# Revision

## Think and Discuss

### The sets of numbers

The set of Counting numbers =  $\{1, 2, 3, \dots\}$

The set of Natural numbers :  $\mathbf{N} = \{0, 1, 2, 3, \dots\} = \text{counting numbers} \cup \{0\}$

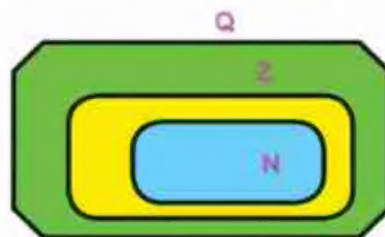
The set of Integers :  $\mathbf{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

The set of Positive integers  $\mathbf{Z}^+ = \{1, 2, 3, \dots\} = \text{Counting numbers}$

The set of Negative integers  $\mathbf{Z}^- = \{-1, -2, -3, \dots\}$

$$\mathbf{Z} = \mathbf{Z}^+ \cup \{0\} \cup \mathbf{Z}^-$$

The set of Rational numbers  $\mathbf{Q} = \{\frac{a}{b} : a, b \in \mathbf{Z}, b \neq 0\}$



$$\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q}$$

### The absolute value of a rational number:

$$|-7| = 7, |3| = 3, |0| = 0, \quad |-\frac{5}{3}| = \frac{5}{3}$$

If  $|a| = 5$  then  $a = \pm 5$





**The Standard form of a rational number is :**

$$a \times 10^n \text{ where } n \in \mathbb{Z}, 1 \leq |a| < 10$$

**For example:-** The standard form of the number  $25.32 \times 10^4$   
 $= 2.532 \times 10^5$

- The standard form of the number  $0.00053 = 5.3 \times 10^{-4}$

**The perfect square rational number:**

It is that positive number which can be written in the form of a square rational number i.e. (rational number)<sup>2</sup>

**Example** 1, 4, 25,  $\frac{9}{16}$ ,  $2\frac{1}{4}$ , ...

**The perfect cube of rational number:**

It is that rational number which can be written in the form of a cube rational number.  
 i.e. (rational number)<sup>3</sup>

**Example** 1, 8, -27, -216,  $\frac{8}{125}$ , ...

**The square root of a perfect square rational number**

- The square root of the positive rational number  $a$  is that number whose square is equal to  $a$ .
- ( $\sqrt{\text{zero}} = \text{zero}$ ) the square root of zero is zero.
- Every perfect square rational number  $a$  has two square roots each one of them is an additive inverse to the other i.e.  $\sqrt{a}$ ,  $-\sqrt{a}$

**Example**  $\frac{16}{25}$  has two square roots:  $\frac{4}{5}$ ,  $-\frac{4}{5}$

- $\sqrt{9}$  means the positive square root of 9 which is equal to 3

$$\sqrt{\left(\frac{a}{b}\right)^2} = \left|\frac{a}{b}\right| \text{ i.e.}$$

$$\sqrt{(-7)^2} = |-7| = 7$$



**Practice**

Complete the following table

Number	Natural Number	Integer	Rational Number
3	✓	✓	✓
-3			
$\frac{3}{5}$			
$\sqrt{\frac{9}{16}}$			
$ 5 - 7 $			



## Revision exercises



- A  $0,2 = \dots\dots\dots$       B  $0,3 = \dots\dots\dots$       C  $25\% = \dots\dots\dots$   
D  $|-0,75| = \dots\dots\dots$       E  $-6 = \dots\dots\dots$       F  $1\frac{1}{4} = \dots\dots\dots$



- A** The solution set of the equation  $x + 5 = |-5|$  in  $\mathbb{N}$  is ..... ( $\{0\}$  or  $\{10\}$  or  $\{-10\}$ ,  $\emptyset$ )
- B** The rational number lies between  $\frac{1}{5}$ ,  $\frac{2}{5}$  is ..... ( $\frac{2}{10}$  or  $\frac{1}{10}$  or  $0.3$  or  $-0.3$ )
- C** The product of the rational number  $\frac{a}{b}$  by its additive inverse is .....
- D**  $|-2| + |-4| + |6| = \dots\dots\dots$
- E**  $\sqrt{a^2} = \dots\dots\dots$
- (zero,  $-\frac{a}{b}$  or  $\frac{a^2}{b^2}$  or  $\frac{-a^2}{b^2}$ )
- (zero,  $|-12|$ ,  $-12$ ,  $6$ )
- ( $a$  or  $-a$  or  $|a|$  or  $\pm a$ )



- A**  $5X + 3 = 20$       **B**  $7X + 11 = 12$   
**C**  $3X + 5 = 1$       **D**  $X + 3 = 7$



- A**  $\sqrt{25 + 144} = \dots\dots\dots$
- B** The standard form of 0 00015 is  $\dots\dots\dots$
- C**  $\sqrt{0.16} + |-0.6| = \dots\dots\dots$
- D**  $2^0 + 2^1 + 2^2 + 2^3 = \dots\dots\dots$
- E** The sum of the two square roots of the number  $2\frac{1}{4} = \dots\dots\dots$
- F**  $\sqrt{0.25} = \dots\dots\dots$





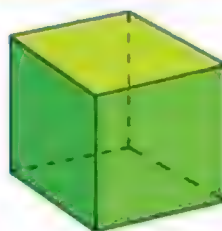
# Unit One

## Lesson One

### The cube root of a rational number

#### Think and Discuss

you have learned that:  
The volume of a cube =  
the length of its side  $\times$  itself  $\times$  itself



#### Complete

The volume of the cube whose side length is equal to 7 cm  
= .....  $\times$  .....  $\times$  ..... = .....  $\text{cm}^3$



#### Let's think

If we have a cube of volume  $125 \text{ cm}^3$ , what is the length of its side?

We search for any three equal numbers of a product equal to 125. Then the number 125 can be factorized into its prime factors

$$125 = 5 \times 5 \times 5$$

$\therefore$  the cube of volume  $125 \text{ cm}^3$  has a side length = 5cm

Therefore, 5 is called the cube root of 125 and it is written as  $\sqrt[3]{125} = 5$ .

125	5
25	5
5	5
1	

#### you will learn how

- To find the cube root of a rational number using factorization.
- To find the cube root of a rational number using the calculator.
- To solve equations that include finding the cube root.
- To solve applications on the cube root of a rational number.

#### Key terms

- Cube root .

The cube root of the rational number  $a$  is that number whose cube is equal to  $a$

- The cube root for the rational number  $a$  is symbolized by  $\sqrt[3]{a}$
- The cube root for a positive rational number is also positive  
Ex:  $\sqrt[3]{125} = 5$
- The cube root for a negative rational number is also negative. Ex:  $\sqrt[3]{-8} = -2$  why ?
- $\sqrt[3]{\text{zero}} = \text{zero}$
- $\sqrt[3]{a^3} = a$



To find the cube root of a perfect cube rational number:

- The number can be factorized into its prime factors..
- A calculator can be used.

**Remark** The perfect cube rational number has one cube root which is also a rational number, why?



### Examples

- 1 Use factorization to find the value of each  $\sqrt[3]{1000}$ ,  $\sqrt[3]{-216}$ ,  $\sqrt[3]{3\frac{3}{8}}$ ; then check your answer using the calculator.

**Solution**

$$\begin{array}{r|l}
 2 & 1000 \\
 \hline
 2 & 500 \\
 2 & 250 \\
 2 & 125 \\
 5 & 25 \\
 5 & 5 \\
 & 1
 \end{array}$$

$$\sqrt[3]{1000} = 5 \times 2 = 10$$

$$\begin{array}{r|l}
 2 & 216 \\
 \hline
 2 & 108 \\
 2 & 54 \\
 3 & 27 \\
 3 & 9 \\
 3 & 3 \\
 & 1
 \end{array}$$

$$\sqrt[3]{-216} = -2 \times 3 = -6$$

$$3\frac{3}{8} = \frac{27}{8}
 \begin{array}{r|l}
 3 & 27 \\
 \hline
 3 & 9 \\
 3 & 3 \\
 & 1
 \end{array}$$

$$\sqrt[3]{3\frac{3}{8}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

Use your calculator to check your answer by pressing on

- 2 Find the length of the radius of a sphere whose volume is equal to  $4851\text{cm}^3$  ( $\pi = \frac{22}{7}$ )

**Solution**

The volume of the sphere =  $\frac{4}{3} \pi r^3$

$$4851 = \frac{4}{3} \times \frac{22}{7} r^3$$

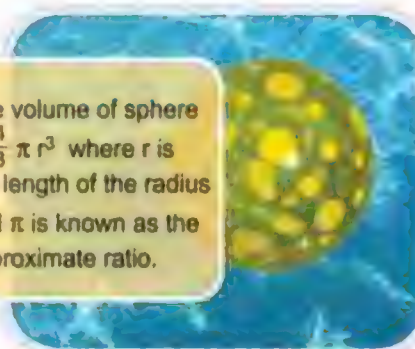
$$r^3 = \frac{4851 \times 3 \times 7}{4 \times 22} = \frac{9261}{8}$$

$$\therefore r^3 = \frac{3^3 \times 7^3}{2^3}$$

$$\therefore r = \sqrt[3]{\frac{3^3 \times 7^3}{2^3}}$$

$$\begin{array}{r|l}
 3 & 9261 \\
 \hline
 3 & 3087 \\
 3 & 1029 \\
 7 & 343 \\
 7 & 49 \\
 7 & 7 \\
 & 1
 \end{array}$$

The volume of sphere =  $\frac{4}{3} \pi r^3$  where  $r$  is the length of the radius and  $\pi$  is known as the approximate ratio.





## Unit 1 : Lesson 1

$$r = \frac{3 \times 7}{2} = \frac{21}{2} = 10.5 \text{ cm}$$

we can use the calculator to find  $\sqrt[3]{\frac{9261}{8}}$  directly.



### Practice

Find the diameter of the sphere whose volume is  $113.04 \text{ cm}^3$  ( $\pi = 3.14$ )



### Example

Solve each of the following equations in Q.

A  $x^3 = 8$

B  $x^3 + 9 = 8$

C  $(x - 2)^3 = 125$

D  $(2x - 1)^3 - 10 = 54$

### Solution

A  $x^3 = 8$

$$x = \sqrt[3]{8} = 2$$

$$\therefore \text{Solution set} = \{2\}$$

B  $x^3 + 9 = 8$

$$x^3 = 8 - 9$$

$$x^3 = -1$$

$$x = \sqrt[3]{-1} = -1$$

$$\therefore \text{Solution set} = \{-1\}$$

C  $(x - 2)^3 = 125$

$$x - 2 = \sqrt[3]{125}$$

$$x - 2 = 5$$

$$x = 7$$

$$\therefore \text{Solution set} = \{7\}$$

D  $(2x - 1)^3 - 10 = 54$

$$(2x - 1)^3 = 64$$

$$2x - 1 = \sqrt[3]{64}$$

$$2x - 1 = 4$$

$$2x = 5$$

$$x = \frac{5}{2}$$

$$\therefore \text{Solution set} = \left\{ \frac{5}{2} \right\}$$



### Practice

Solve the following equations in Q:  $(x + 1)^3 = 27$  ,  $(x + 1)^3 = -27$



## Exercises (1-1)

1 Fill in the following table:

Number a	8	125	-27	.....	$3\frac{3}{8}$	$-\frac{8}{125}$	.....	.....
$\sqrt[3]{a}$	.....	.....	.....	-10	.....	.....	6	-4

2  Complete

A  $\sqrt[3]{-125} = \dots\dots\dots$       B  $\sqrt[3]{343} = \dots\dots\dots$       C  $\sqrt[3]{8} + \sqrt[3]{-8} = \dots\dots\dots$   
 D  $\sqrt[3]{0.001} = \dots\dots\dots$       E  $\sqrt[3]{27} - \sqrt[3]{64} = \dots\dots\dots$       F  $\sqrt[3]{a^3} = \dots\dots\dots$

3  Choose the correct answer from the Parentheses:

A  $\sqrt[3]{(-8)^2} = \dots\dots\dots$  (2 or -2 or 4 or -4)  
 B  $\sqrt[3]{25} - \sqrt[3]{-125} = \dots\dots\dots$  (10 or 0 or 5 or  $\pm 5$ )  
 C  $\sqrt[3]{3\frac{3}{8}} + \sqrt[3]{0.25} = \dots\dots\dots$  ( $\frac{3}{2}$  or  $\frac{1}{2}$  or 2 or -2)  
 D  $\sqrt[3]{1000} \times \sqrt[3]{-0.008} = \dots\dots\dots$  ( $\frac{1}{2}$  or 10 or 2 or -2)  
 E The lateral area of a cube whose volume is  $216 \text{ cm}^3 = \dots\dots\dots \text{cm}^2$  (36 or 6 or 144 or 216)  
 F  $\sqrt[3]{x^6} = \sqrt{\dots\dots\dots}$  ( $x^3$  or  $x^2$  or  $x$  or  $x^4$ )  
 E  $\sqrt[3]{-27} + \sqrt[3]{12\frac{1}{4}} + \sqrt[3]{0.125} = \dots\dots\dots$  (1 or 0 or -1 or  $\frac{11}{2}$ )

4  Find the value of x in each of following cases:

A  $\sqrt[3]{x} = 5$       B  $\sqrt[3]{x} = -\frac{1}{2}$       C  $\sqrt[3]{x} = -\sqrt[3]{4}$   
 D  $x^3 = -8$       E  $x^3 - 125 = 0$       F  $x^2 = 64$

5  Find the solution set for each of the following equations in Q:

A  $x^3 + 27 = 0$       B  $8x^3 + 7 = 8$   
 C  $(x + 3)^3 = 343$       D  $(5x - 2)^3 + 10 = 18$

6 Application problems

A Find the side length of a cube vessel with capacity of one liter.

B Find the diameter length of a sphere whose volume =  $\frac{1372}{81} \pi$  cubic unit  
(Use  $\pi = \frac{22}{7}$ ) (volume of a sphere =  $\frac{4}{3} \pi r^3$ )





The set of Irrational  
numbers  $Q'$ 

## Think and Discuss

**you have learned that:** A rational number is that number which can be put in the form:

$$\frac{a}{b} : \text{where } a, b \in \mathbb{Z}, b \neq 0$$

**for example:** when solving the equation  $4x^2 = 25$

$$\text{then } x^2 = \frac{25}{4} \quad \therefore x = \pm \frac{5}{2}$$

**Remark** Each of  $\frac{5}{2}$ ,  $-\frac{5}{2}$  is a rational number.

However, there are many numbers which can not be put in the form  $\frac{a}{b}$  where  $a, b \in \mathbb{Z}, b \neq 0$

**for example :** when solving the equation  $X^2 = 2$ , we can not find any rational number whose square is equal to 2

**you will learn how how**

To define the set of irrational numbers.

**key terms**

Irrational number

**The irrational number**

It is that number which can not be put in the form  $\frac{a}{b}$  where  $a, b \in \mathbb{Z}, b \neq 0$

the following are examples to irrational numbers.

**First :** the square roots of the positive numbers which are not perfect squares

$$\text{Ex : } \sqrt{2}, \sqrt{5}, -\sqrt{6}, \sqrt{7}$$

**Second:** the cube roots of those numbers that are not perfect cubes

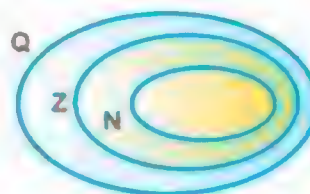
$$\text{Ex : } \sqrt[3]{4}, \sqrt[3]{-2}, \sqrt[3]{11}, \dots$$

**Third:** the pi  $\pi$  (the approximation ratio)

Where it is impossible to find any exact value for any of the previous number. why?



Those numbers and others form a set which is called the set of irrational numbers which is denoted by the symbol  $Q'$ .



$$Q \cap Q' = \emptyset$$



**Think** : is  $\sqrt[3]{-1}$  an irrational number? why?

## Exercises (1-2)

1



**Complete** : using the symbol  $Q$  or  $Q'$ .

A  $5 \in \dots\dots\dots$

B  $\sqrt{10} \in \dots\dots\dots$

C  $0 \in \dots\dots\dots$

D  $-0.7 \in \dots\dots\dots$

E  $\sqrt{8} \in \dots\dots\dots$

F  $\sqrt[3]{6} \in \dots\dots\dots$

G  $\sqrt{-9} \in \dots\dots\dots$

H  $\pi \in \dots\dots\dots$

2

Put (✓) on the true sentence and (X) on the false sentence

A  $2.3 \times 10^5 \in Q$  ( ) B  $|-5| \in Q'$  ( ) C  $\frac{\text{zero}}{5} \in Q$  ( )

D  $\sqrt{-4} \in Q'$  ( ) E  $\sqrt{1000} \in Q$  ( ) F  $\sqrt{7} > 3$  ( )

G  $\sqrt{10} > 2$  ( ) H  $\sqrt{20} > \sqrt{9}$  ( )

I The side length of a square whose surface area =  $6\text{cm}^2$  is a rational number ( )

3

Choose the correct answer:

A The surface area of a square whose side length is  $\sqrt{3} = \dots\dots\dots$

( $4\sqrt{3}$  or 9 or 3 or 6)

B The irrational number lies between 3 and 4 is  $\dots\dots\dots$

( $3.5$  or  $\frac{1}{8}$  or  $\sqrt{13}$  or  $\sqrt{20}$ )

C The irrational number lies between -2 and -1 is  $\dots\dots\dots$

(-3 or -1  $\frac{1}{2}$  or  $-\sqrt{3}$  or  $\sqrt{2}$ )





# Unit One

## Lesson Three

### Finding the approximate value of an Irrational number

#### Think and discuss

Can you find the two rational numbers which the irrational number  $\sqrt{2}$  is located between them.

#### Remark

$\sqrt{2}$  is between  $\sqrt{1}$ ,  $\sqrt{4}$  i.e.  $1 < \sqrt{2} < 2$   
i.e.  $\sqrt{2} = 1 + \text{a decimal fraction}$

To find the approximate value of  $\sqrt{2}$ . We check the values of the following numbers:

$$(1.1)^2 = 1.21, (1.2)^2 = 1.44, (1.3)^2 = 1.69,$$

$$(1.4)^2 = 1.96, (1.5)^2 = 2.25$$

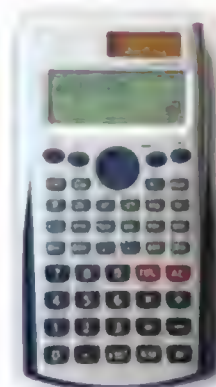
$$\therefore 1.96 < 2 < 2.25$$

$$\therefore 1.4 < \sqrt{2} < 1.5$$

i.e.  $\sqrt{2} = 1.4 + \text{a decimal fraction}$

$$\text{i.e. } 1.41 < \sqrt{2} < 1.42$$

Use the calculator to check your answer.



#### You will learn how

- To find the approximate value for an irrational number
- To represent an irrational number on the number line.
- To solve equations in  $\mathbb{Q}$

Representing the irrational number on the number line.

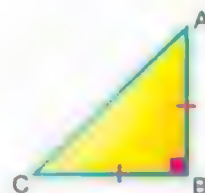
How can the point represents  $\sqrt{2}$  be located on the number line?

If we draw the right triangle ABC at B which is an isosceles triangle also.

where  $AB = BC = \text{one unit of length}$

$$\text{Then } (AC)^2 = (AB)^2 + (BC)^2 = 1^2 + 1^2 = 2$$

$$\therefore AC = \sqrt{2} \text{ unit of length.}$$

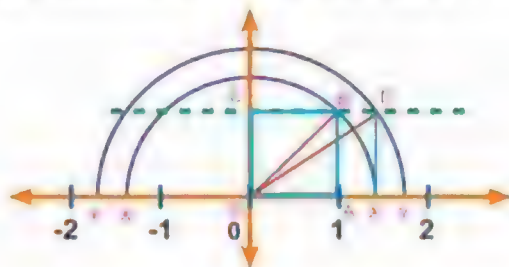


- draw the number line and place the sharp point of the compasses at point O, then adjust the compasses to a length that is equal to  $\overline{AC}$  and draw an arc that intersects the number line on the right of o and at the point X, where that point represents  $\sqrt{2}$
- Using the same length, we can label the point  $X^*$  which represent  $-\sqrt{2}$  where  $X^*$  is on the left of the point o.



**Think** : Label the point which represents  $3 + \sqrt{2}$  on the number line.

**Activity** : Draw the square O A B C whose side length is equal to one unit of length.



The length of its diagonal =  $\sqrt{1+1} = \sqrt{2}$  unit of length  
 $\therefore OB = \sqrt{2}$

- Place the sharp point of the compasses at point O and draw a semi-circle whose diameter = the length of  $\overline{OB} = \sqrt{2}$ .
- $\overleftrightarrow{OA} \cap$  the semi-circle =  $\{X, X^*\}$  where X represents the number  $\sqrt{2}$ ,  $x^*$  represents the number  $-\sqrt{2}$ .
- Draw  $\overline{XD} \parallel \overline{AB}$  and intersects  $\overleftrightarrow{CB}$  at D  
 $(OD)^2 = (OX)^2 + (XD)^2 = (\sqrt{2})^2 + (1)^2 = 3$   
 $\therefore OD = \sqrt{3}$
- Place the sharp point of the compasses at point O and adjust it to a length which is equal to the length of  $\overline{OD}$ , then draw semi-circle that intersects with  $\overleftrightarrow{OA}$  at points Y,  $Y^*$   
 $\therefore OY = \sqrt{3}$  i.e. point Y represents  $\sqrt{3}$ , while point  $Y^*$  represents  $-\sqrt{3}$
- Continue using the same method to represent  $\sqrt{4}, \sqrt{5}, \sqrt{6}, \dots$   
also  $-\sqrt{4}, -\sqrt{5}, -\sqrt{6}, \dots$







## Practice

## 1 Find :

- A Two consecutive integers that  $\sqrt{5}$  lies between them.
- B Two consecutive integers that  $\sqrt{12}$  lies between them.
- C Two consecutive integers that  $\sqrt[3]{10}$  lies between them.
- D Two consecutive integers that  $\sqrt{-20}$  lies between them.

## 2 Prove that :

- A  $\sqrt{3}$  lies between 1.7 , 1.8 .
- B  $\sqrt[3]{15}$  lies between 2.4 , 2.5 .

3 Find the value of  $\sqrt{11}$  to the nearest hundredth.4 Find the value of  $\sqrt[3]{2}$  to the nearest tenth.5 Draw the number line and label the point which represents the irrational number  $\sqrt{3}$ .6 Draw the number line and label the point which represents the irrational number  $1 + \sqrt{2}$ 

## Example (1)

Find the solutions set for each of the following equations in  $\mathbb{Q}$ :

A  $x^2 = 2$

B  $x^3 = 5$

C  $\frac{4}{3}x^2 = 1$

D  $0.001 x^3 = -8$

## Solution

A  $x^2 = 2$

$\therefore x = \pm \sqrt{2}$  Solution set =  $\{-\sqrt{2}, \sqrt{2}\}$

B  $x^3 = 5$

$\therefore x = \sqrt[3]{5}$  Solution set =  $\{\sqrt[3]{5}\}$

C  $\frac{4}{3}x^2 = 1$

$\therefore \frac{3}{4} \times \frac{4}{3}x^2 = \frac{3}{4} \times 1$

$x^2 = \frac{3}{4}$

$\therefore x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$

Solution set =  $\{-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}$



$$D \quad 0.001 x^3 = -8$$

$$x^3 = -\frac{8}{0.001} = -8000$$

$$\therefore x = \sqrt[3]{-8000}$$

$$= -20 \in Q$$

The solution set in  $Q' = \emptyset$



### Example (2)



**Find** the length of each of the side and the diagonal of a square whose area is  $7\text{cm}^2$ .

#### Solution

Let the length of the side be  $x$  cm,

then the area  $= x \times x = x^2$

Where  $L$  is the square diagonal length

$$x^2 = 7$$

$$\therefore x = \pm\sqrt{7} \text{ cm}$$

$$\therefore x = \sqrt{7} \text{ cm why?}$$

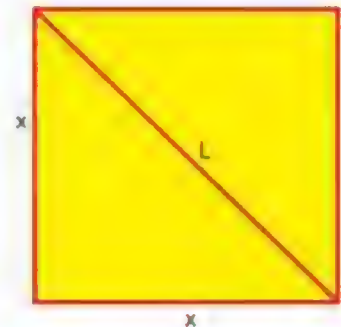
To find the diagonal of the square: use pythagorean theorem

$L^2 = x^2 + x^2$  Where  $L$  is the square diagonal length.

$$\therefore L^2 = 14$$

$$\therefore L = \pm\sqrt{14} \text{ cm}$$

$$\therefore L = \sqrt{14} \text{ cm why?}$$



### Example (3)



**Find** : the circumference of a circle whose area is  $3\pi \text{ cm}^2$

#### Solution

The area of the circle  $= \pi r^2$

$$3\pi = \pi r^2$$

$$\therefore r^2 = 3$$

$$r = \sqrt{3} \text{ cm} \quad \text{or } r = -\sqrt{3} \text{ cm (refused)}$$

$$\text{the circumference} = 2\pi r = 2\pi \times \sqrt{3} = 2\sqrt{3}\pi \text{ cm.}$$






## Exercises (1-3)

- 1 Circle the irrational number in each of the following:

$$\sqrt{3}, -0,2, \sqrt[3]{-1}, 0, \sqrt[3]{9}, -\sqrt{\frac{4}{25}}$$

- 2  **Find** the value of X in each of the following cases and determine whether  $x \in \mathbb{Q}$  or  $x \in \mathbb{Q}^*$

A  $4x^2 = 9$

B  $2x^2 = 6$


C  $x^3 = 125$

D  $x^3 = 10$

E  $(x - 1)^2 = 4$

F  $(x - 2)^3 = 1$

- 3  **Find** the approximate value for  $\sqrt{10}$ , and check your answer using the calculator.

- 4  **Think** : If x is an integer, find the value of X in each the following cases:

A  $x < \sqrt{7} < x + 1$


B  $x < \sqrt{80} < x + 1$

C  $x < \sqrt{125} < x + 1$

D  $x < \sqrt[3]{5} < x + 1$

E  $x < \sqrt[3]{30} < x + 1$

F  $x < \sqrt[3]{100} < x + 1$

- 5  **Choose** the correct answer :

A the irrational number lies between 2 and 3 is. ( $\sqrt{10}$  or  $\sqrt{7}$  or 2.5 or  $\sqrt{3}$ ).

B  $\sqrt{10} \approx \dots\dots\dots$  (2.99 or 3.71 or 3 or -3.2) .

C The nearest integer to  $\sqrt{25}$  is  $\dots\dots\dots$  (5 or 3 or 2 or 12.5) .

D The square whose area is  $10 \text{ cm}^2$ , its side length is  $\dots\dots\dots$  cm  
(5 or -5 or  $\sqrt{10}$  or  $-\sqrt{10}$ ).

E The cube whose volume is  $64 \text{ cm}^3$  its side length is  $\dots\dots\dots$  cm (8 or 4 or 16 or 64).

- 6 Draw the number line and label point A which represents  $\sqrt{2}$  and  
Label point B which represents  $1 + \sqrt{2}$   
, Label point C which represents  $1 - \sqrt{2}$

- 7 Draw the right triangle ABC at B where  $AB = 2\text{cm}$ ,  $BC = 3\text{cm}$ , then use the figure to label a point which represents  $\sqrt{13}$ , and a point which represents  $-\sqrt{13}$  on the number line.



# Unit One

## Lesson

4

## The set of the Real numbers R

### Think and Discuss

#### You will learn how

- To define the set of real numbers (R).
- To define the relation among sets of N, Z, Q, Q', R

#### Key terms

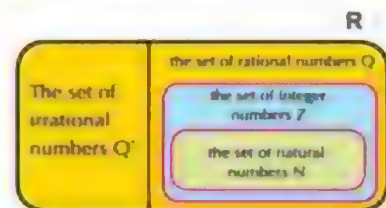
- A real number.

You have learned the set of rational numbers (Q), you have also found that there are other numbers that form the set of irrational number  $Q'$  such as  $\sqrt{2}$ ,  $\sqrt[3]{2}$ ,  $\pi$ ,... However, the union of these two sets forms a new set called the set of the real numbers, and it is denoted by the symbol R

$$R = Q \cup Q'$$

Look at the opposite Venn diagram, you find that:

- 1  $R = Q \cup Q'$
- 2 Any natural, integer, rational or irrational number is a real number



$$N \subset Z \subset Q \subset R \quad \text{and so is } Q' \subset R$$



**Think** Give examples from your own to some real numbers which are rational or irrational numbers.

- 3 Every real number is represented by one point on the number line.



**First:** zero is represented by the origin O.

**Second:** the positive real numbers are represented by all the points On the number line that are located on the right side of O

**Third:** the negative real numbers are represented by all the points on the number line that are located on the left side of O



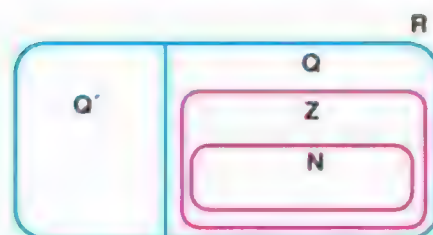




## Practice

- 1 Put each of the following numbers in its suitable place on the opposite venn diagram.

$\frac{1}{2}, -4, 9, \sqrt{5}, 0, 6, \frac{7}{9}, \sqrt[3]{-2}, \sqrt{16}, 0, 5$



- 2 Label point A on the number line which represents  $\sqrt[3]{-8}$ , and point B which represents  $\sqrt{9}$ , then find the length of  $\overline{AB}$ .



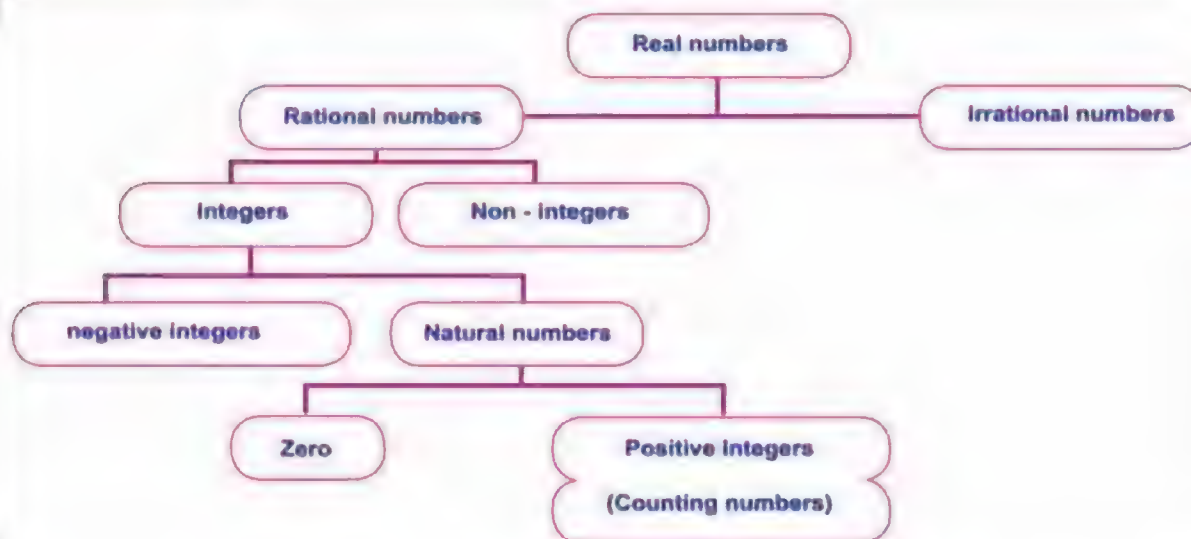
- 3 State if each sentence is true or false:

- A Every natural number is a positive real number.  
B Every integer is a real number.

### Remark

$\sqrt[3]{-1} = -1$  because  $-1 \times -1 \times -1 = -1$

While  $\sqrt{-1} \notin \mathbb{R}$  because there is no real number if multiplied by it self, the product is  $-1$ .



**Discuss** with your teacher and classmates: Are there any non- Real number?



## Exercises (1 - 4)

- 1 Study the previous chart and answer the following by placing (✓) on the true sentence and (X) on the false sentence:

- A Every natural number is an integer. ( )
- B Zero  $\in$  The set of rational numbers ( )
- C  $\mathbb{Z} = \mathbb{Z}^+ \cup \mathbb{Z}^-$  ( )
- D Any non-integer number is a rational number ( )

- 2 Complete the following table by placing (✓) in the suitable place as shown in the first case:

Number	natural number	integer number	rational number	irrational number	real number
-5	X	✓	✓	X	✓
$\sqrt{2}$					
$1\frac{1}{2}$					
$\sqrt[3]{9}$					
$ -2 $					
$-\sqrt{4}$					
$\frac{5}{2}$					
0,3					
$\sqrt{-1}$					





# Unit One

## Lesson Five

### Ordering numbers at R

#### Think and Discuss

If A, B are two points that belong to the straight line L, and we determined a certain direction as shown by the arrow; then we can say that:



- The point B follows the point A. i.e on its right hand side.
- The point A precedes the point B. i.e on its left hand side.

The same applies for all the points on the straight line.

However, If we know that every point on the straight line represent a real number. We can say that :

**the set of real number is an ordered set.:**

#### The properties of order:

- 1 If  $x, y$  are two real numbers represented on the number line by the two points A, B respectively, the ordering relation can be one of the following three cases:

<b>A is congruent to B so <math>x = y</math></b>	<b>A follows B so <math>x &gt; y</math></b>	<b>A precedes B so <math>x &lt; y</math></b>

- 2 If  $x$  is a real number represented by the point A on the number line while O is the origin point which represents the zero, then the ordering relation can be one of the following three cases.

<b>A is congruent to O <math>\therefore x = 0</math></b>	<b>A is on the right of O <math>\therefore x &gt; 0</math> then <math>x</math> is a positive real number.</b>	<b>A is on the left of O <math>\therefore x &lt; 0</math> then <math>x</math> is a negative real number.</b>

#### You will learn how

- ☞ To define the ordering relation in R.

#### Key terms

- ☞ ordering relation
- ☞ more than.
- ☞ Less than
- ☞ Equal to
- ☞ Ascending order
- ☞ Descending order .





The set of the positive real numbers:  $R^+ = \{x : x \in R, x > 0\}$

The set of the negative real numbers:  $R^- = \{x : x \in R, x < 0\}$

$$R = R^+ \cup \{0\} \cup R^-$$

**Remark:**

The set of non-negative real numbers =  $R^+ \cup \{0\} = \{x : x \geq 0, x \in R\}$

The set of the non - positive real numbers =  $R^- \cup \{0\} = \{x : x \leq 0, x \in R\}$



**Example:**

Arrange the following numbers ascendingly  $\sqrt{27}$ ,  $-\sqrt{45}$ ,  $\sqrt{20}$ ,  $6$ ,  $0$ ,  $\sqrt[3]{-1}$

**Solution**

$$6 = \sqrt{36}, \sqrt[3]{-1} = -1 = -\sqrt{1}$$

The ascending order is from the smallest to the greatest.

$$-\sqrt{45}, -\sqrt{1}, 0, \sqrt{20}, \sqrt{27}, \sqrt{36}$$

$$\text{i.e. } -\sqrt{45}, \sqrt[3]{-1}, 0, \sqrt{20}, \sqrt{27}, 6.$$

## Exercises (1-5)

- 1 Arrange the following number descendingly :  $\sqrt{62}$ ,  $8$ ,  $-\sqrt{50}$ ,  $\sqrt{70}$
- 2 If  $x \in R$ , state whether  $x$  is a positive or negative or anything else in each of the following.
 

A  $x > 0$ 
B  $x < 0$ 
C  $x > |-5|$
- 3 Prove that  $\sqrt{3}$  lies between 1.7, 1.8, then represent  $\sqrt{3}$ , 1.7 and 1.8 on the number line.
- 4 Find the side length in a square whose area is  $5 \text{ cm}^2$ , is the side length a rational number?
- 5 Find the side length in a cube whose volume is  $1.728 \text{ cm}^3$ , is the side length a rational number?
- 6 Put the suitable notation(  $>$  or  $<$  or  $=$  )
 

A  $\sqrt{5} \dots\dots 2$

B  $\sqrt{7} \dots\dots 2,6$

C  $\sqrt[3]{-24} \dots\dots -2$

D  $1 + \sqrt{2} \dots\dots \sqrt{3}$

E  $\sqrt[3]{8} \dots\dots \sqrt{4}$

F  $3 - \sqrt{5} \dots\dots \sqrt{-1}$





## Intervals

## Think and Discuss

**Interval** is a subset of the set of real numbers

first: the limited intervals

If  $a, b \in \mathbb{R}$ ,  $a < b$ , then we can define each of:

The closed interval  
[a , b]

$$[a, b] = \{ x : a \leq x \leq b, x \in \mathbb{R} \}$$



$[a, b] \subset \mathbb{R}$  in which the elements are  $a$ ,  $b$  and all the real numbers between them.

When we draw that interval, we put a shaded circle at each of the two points  $a$  and  $b$  then, we shade that area between them on the number line.

The open interval  
)a , b[

$$]a, b[ = \{ x : a < x < b, x \in \mathbb{R} \}$$



$]a, b[ \subset \mathbb{R}$  in which the elements are all the real numbers between the two numbers  $a$ ,  $b$

When we draw that interval, we put an unshaded circle at each of the two points which represent the two numbers  $a$  and  $b$  then, we shade that area between them on the number line.

## You will learn how

- To define limited intervals.
- To define unlimited intervals.
- To recognize the operations on intervals .

## key terms

- Limited interval
- closed interval
- open interval
- half- open interval
- unlimited interval
- union
- intersection
- difference
- complement

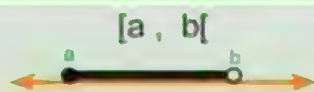


## Practice

Write down each of  $[3, 5]$ ,  $]3, 5[$  using the description method then represent them on the number line.

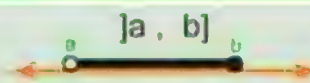


## Half openor (half closed) intervals



$$[a, b[ = \{x : a \leq x < b, x \in \mathbb{R}\}$$

$[a, b[ \subset \mathbb{R}$  where its elements are the number  $a$  and all the numbers between  $a$  and  $b$ .



$$]a, b] = \{x : a < x \leq b, x \in \mathbb{R}\}$$

$]a, b] \subset \mathbb{R}$  where its elements are the number  $b$  and all the number between  $a$  and  $b$ .



### Practice

Write down each of the two intervals:  $[3, 5[$ ,  $]3, 5]$  using the description method, then represent them on the number line.



### Examples :

Represent each of the following intervals on the number line:  $[-1, 4]$ ,  $] -1, 4[$ ,  $] -1, 4]$ ,  $\{-1, 4\}$

### Solution



Discuss with your teacher and your classmates whether the interval is a finite or an infinite set.



### Practice

1. Write down the following sets in the form of intervals, then represent them on the number line:

A  $X = \{x : 2 < x < 5, x \in \mathbb{R}\}$

B  $X = \{x : -2 \leq x < 3, x \in \mathbb{R}\}$

C  $X = \{x : 0 \leq x \leq 4, x \in \mathbb{R}\}$

D  $X = \{x : -3 < x \leq -1, x \in \mathbb{R}\}$





2 Put The suitable symbol  $\in$  or  $\notin$  to make each sentence true.

- A  $3 \dots [-1, 3[$       B  $-2 \dots ]-1, 3[$       C  $\frac{1}{2} \dots ]0, 1[$   
D  $\sqrt{2} \dots [1, 2]$       E  $4 \dots [0, 5[$       F  $\sqrt[3]{-8} \dots [-1, 2]$   
G  $|-5| \dots [4, 6[$       H  $2.3 \times 10^{-5} \dots ]0, 1[$

3 Write down the interval represented by each of the following figures::



### Second: The unlimited intervals

**You know that:** If the number line of real numbers is expanded on its two direction, we get more positive real numbers at the right direction and more negative real number at the left direction such all those numbers are located on that line.

- The symbol  $(\infty)$  is read (infinity) and it is more than any imagined real number,  $\infty \notin \mathbb{R}$
- The symbol  $(-\infty)$  is read (negative infinity) and it is less than any imagined real number,  $-\infty \notin \mathbb{R}$
- The two symbols  $\infty, -\infty$  can not be represented by any points on the number line and they are expansions to the number line at its two directions.



If  $a$  is a real number, then we can define the following unlimited intervals:

The interval  $[a, \infty[$   
 $[a, \infty[ = \{x : x \geq a, x \in \mathbb{R}\}$



That interval represents the number  $a$  and all the real numbers which are more than  $a$

The interval  $]-\infty, a]$   
 $]-\infty, a] = \{x : x \leq a, x \in \mathbb{R}\}$



That interval represents the number  $a$  and all the real number which are less than  $a$ .

Write down each of the following intervals  $[3, \infty[, ]-\infty, 3]$  using the description method, then represent them on the number line.



the interval  $]a, \infty[$

$$]a, \infty[ = \{x : x > a, x \in \mathbb{R}\}$$



That interval represents all the real number which are more than a

the interval  $]-\infty, a[$

$$]-\infty, a[ = \{x : x < a, x \in \mathbb{R}\}$$



that interval represents all the real numbers which are less than a



**Write** down the two intervals  $]3, \infty[$  ,  $]-\infty, 3[$  using the description method , then represent them on the number line

**Remark :**

The set of real numbers ( $\mathbb{R}$ ) can be represented in the form of the interval  $]-\infty, \infty[$

The set of the positive real numbers  $\mathbb{R}^+ = ]0, \infty[$

The set of the negative real numbers  $\mathbb{R}^- = ]-\infty, 0[$

The set of non-negative real numbers =  $[0, \infty[$

The set of non-positive real numbers =  $]-\infty, 0]$



**Practice**

1



**Write** down the following sets in the form of intervals, then represent them on the number line.

A  $X = \{x : x \geq 2, x \in \mathbb{R}\}$

B  $X = \{x : x < 3, x \in \mathbb{R}\}$

C  $X = \{x : x > -7, x \in \mathbb{R}\}$

D  $X = \{x : x \leq \sqrt[3]{-8}, x \in \mathbb{R}\}$

E the set of all the real numbers more than  $|-3|$

2



**Put** the suitable symbol  $\in$  or  $\notin$  or  $\subset$  or  $\not\subset$  To make each statement true:

A  $3 \dots ]-\infty, 4[$

B  $[1, 2] \dots ]-1, \infty[$

C  $-5 \dots ]-\infty, -6[$

D  $]0, 2[ \dots ]0, \infty[$

E  $3 \times 10^{10} \dots ]3, \infty[$

G  $[-3, 1] \dots [2, \infty[$





### Operations on intervals

Since all the intervals are subsets of the set of the real number  $\mathbb{R}$ , The operations of union, intersection, difference and complement can be applied on the intervals. The graphical representation to the intervals on the number line contributes to determine and verify the result of any operation. This can be clarified from the following examples:



#### Examples

1 If  $X = [-2, 3]$ ,  $Y = [1, 5[$ , find the following using the number line:

A  $X \cap Y$

B  $X \cup Y$

#### Solution

A  $X \cap Y = [-2, 3] \cap [1, 5[ = [1, 3]$

B  $X \cup Y = [-2, 3] \cup [1, 5[ = [-2, 5[$



2 If  $M = [2, \infty[$ ,  $J = ]-2, 3[$ , find the following using the number line:

A  $M - J$

B  $M \cap J$

C  $M \cup J$

D  $J \cup \{2, 3\}$

E  $M^c$

F  $J^c$

#### Solution

A  $M - J = [2, \infty[ - ]-2, 3[ = [3, \infty[$

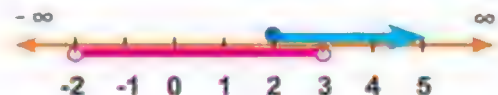
B  $M \cap J = [2, \infty[ \cap ]-2, 3[ = [2, 3[$

C  $M \cup J = [2, \infty[ \cup ]-2, 3[ = ]-2, \infty[$

D  $J \cup \{2, 3\} = ]-2, 3[ \cup \{2, 3\} = ]-2, 3]$

E  $M^c = ]-\infty, 2[$

F  $J^c = ]-\infty, -2] \cup [3, \infty[$



#### Practice



Put (✓) on the true sentence and (X) on the false sentence:

A  $[-2, 5] - \{2, 5\} = ]-2, 5[$

D  $[-1, 3] \cap [1, 4[ = [1, 3]$

B  $] -1, 3[ \cup \{-1, 0\} = [-1, 0]$

E  $[-2, 5[ \cup \{1, 5\} = [-2, 5]$




C  $[2, 5] - \{5\} = [2, 5[$

F  $[5, \infty[ - ]-\infty, 5] = ]5, \infty[$



## Exercises (1-6)

1  **Complete** the following table as shown in the first example:

Interval	Representation by using the description method	Graphical representation on the number line
$[-1, 2]$	$\{x : -1 \leq x \leq 2, x \in \mathbb{R}\}$	
$[1, 3[$		
$]-\infty, 2]$		
	$\{x : 0 < x \leq 3, x \in \mathbb{R}\}$	
	$\{x : x > -1, x \in \mathbb{R}\}$	
		
		
$]1, 5[$		
	$\{x : x > 0, x \in \mathbb{R}\}$	

2  **Complete** using  $\in$  or  $\notin$ :

- A  $3 \dots [2, 3]$       D  $\sqrt{9} \dots ]-3, \infty[$   
 B  $\sqrt{-1} \dots ]-\infty, 1[$       E  $|-2| \dots [2, \infty[$   
 C  $2 \dots (1, 7)$       F  $1.3 \times 10^{-5} * \dots \mathbb{R}$

3  **Choose** the correct answer :

- A  $[2, 7] - \{2, 7\} = \dots$  ( $[1, 6]$  or  $\emptyset$  or  $]2, 7[$  or  $\{0\}$ )  
 B  $[0, 5] \cup [3, 8] = \dots$  ( $]3, 5]$  or  $[3, 5]$  or  $[0, 8]$  or  $[0, 8[$ )  
 C  $[1, 5] \cap ]-2, 3] = \dots$  ( $\{1, 3\}$  or  $]1, 3[$  or  $[1, 3]$  or  $[1, 3[$ )  
 D  $] -1, 2[ - [1, 4] = \dots$  ( $] -1, 1[$  or  $\{-1, 1\}$  or  $] -1, 1]$  or  $[-1, 1])$

4 If  $X = [-1, 4]$ ,  $Y = [3, \infty[$ ,  $Z = \{3, 4\}$ , find each of the following using the number line:

- A  $X \cup Y$       B  $X \cap Y$       C  $X - Y$       D  $X - Z$   
 E  $Y \cap Z$       F  $Y - X$       G  $X'$       H  $Y'$





# Operations on the real numbers

## Unit One

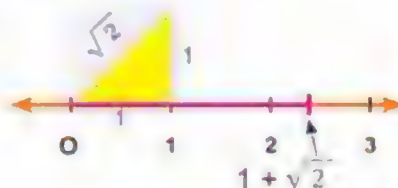
### Lesson Seven

#### Think and Discuss

**First: The properties of adding the real numbers :**

You have determined the location of the point X which represents the number  $1 + \sqrt{2}$  on the number line. Since it represents the sum of the two real numbers 1 and  $\sqrt{2}$  then the sum of every two real numbers is a real number.

i.e, the set of the real numbers R is closed under the operation of addition.



**the closure property**

If  $a \in R$ ,  $b \in R$  then  $(a + b) \in R$

**for example :** each of  $2 + 3$ ,  $1 + \sqrt{2}$ ,  $-2 + \sqrt{5}$  and  $2 + \sqrt[3]{3}$  are real numbers.

**The commutative property**

If  $a \in R$ ,  $b \in R$  then  $a + b = b + a$

**for example :**  $2 + \sqrt{3} = \sqrt{3} + 2$ ,  $3 - \sqrt{5} = -\sqrt{5} + 3$

**The associative property**

If  $a \in R$ ,  $b \in R$ ,  $c \in R$ .

then  $(a + b) + c = a + (b + c) = a + b + c$

**for example :**  $(3 + \sqrt{2}) + 5 = 3 + (\sqrt{2} + 5)$  associative property  
 $= 3 + (5 + \sqrt{2})$  commutative property  
 $= 3 + 5 + \sqrt{2}$  associative property  
 $= 8 + \sqrt{2}$

#### You will learn how

- ✎ To solve operations on the real numbers .
- ✎ To define the properties of operations on the real numbers .

#### key terms

- ✎ Closure property.
- ✎ Commutative property.
- ✎ associative property.
- ✎ Additive neutral.
- ✎ Additive inverse.
- ✎ multiplicative neutral.
- ✎ multiplicative inverse.
- ✎ distribution of multiplication on addition or subtraction.



Zero is the additive neutral element:

If  $a \in \mathbb{R}$  then  $a + 0 = 0 + a = a$

for example :  $\sqrt{5} + 0 = 0 + \sqrt{5} = \sqrt{5}$  ,  $-\sqrt{4} + 0 = 0 + (-\sqrt{4}) = -\sqrt{4}$

Each real number has an additive inverse:

For a number  $a \in \mathbb{R}$  there is  $-a \in \mathbb{R}$

where  $a + (-a) = (-a) + a = \text{zero}$

for example  $\sqrt{3} \in \mathbb{R}$  , has additive inverse  $(-\sqrt{3}) \in \mathbb{R}$  where  
 $\sqrt{3} + (-\sqrt{3}) = (-\sqrt{3}) + \sqrt{3} = \text{zero}$ .



### Practice

1



Complete the following to have a true sentence:

- A  $\sqrt{2} + 5 = 5 + \dots$
- B  $\sqrt{11} + (-\sqrt{11}) = \dots$
- C  $7 + \sqrt{3} = 5 + (\dots + \dots)$
- D the additive inverse for  $\sqrt[3]{8}$  is  $\dots$
- E the additive inverse for  $(1 - \sqrt{2})$  is  $\dots$
- F  $\sqrt{3} + (-\sqrt{3}) = \dots$
- G  $7 + \sqrt{5} - 3 = \dots$
- H  $(4 + \sqrt{7}) + (3 - \sqrt{7}) = \dots$
- I If  $a \in \mathbb{R}$ ,  $b \in \mathbb{R}$ , then  $a - b$  means the sum of the number  $a$  and  $\dots$  of the number  $b$ .
- J If  $a \in \mathbb{N}$ ,  $b \in \mathbb{Q}$ ,  $c \in \mathbb{R}$ , then  $(a + b + c) \in \dots$

2

Discuss the following with your teacher and classmates, then give examples:

- A Is subtraction a commutative operation in  $\mathbb{R}$ ?
- B Is subtraction an associative operation in  $\mathbb{R}$ ?





Second: The properties of multiplying the real numbers

**The closure property** If  $a \in \mathbb{R}, b \in \mathbb{R}$  then  $a \times b \in \mathbb{R}$

the set of real number is closed under the operation of multiplication.

i.e the product of multiplying every two real number is a real number.

**for example** :  $5 \times \sqrt{2} = 5\sqrt{2} \in \mathbb{R}, \sqrt{3} \times \sqrt{3} = 3 \in \mathbb{R}$

$$-2 \times \sqrt[3]{5} = -2\sqrt[3]{5} \in \mathbb{R}, \frac{2}{3} \times \pi = \frac{2}{3}\pi \in \mathbb{R}$$

$$2\sqrt{3} \times \sqrt{3} = 6 \in \mathbb{R}, 2\sqrt{3} \times 5 = 10\sqrt{3} \in \mathbb{R}$$

**Commutative property** If  $a \in \mathbb{R}$  and  $b \in \mathbb{R}$ , then  $a \cdot b = b \cdot a$

**for example** :  $\sqrt{2} \times 3 = 3 \times \sqrt{2} = 3\sqrt{2}$

**The associative property** If  $a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}$ , then :

$$(a \cdot b) \cdot c = a \cdot (b \cdot c) = a \cdot b \cdot c$$

**for example** :  $\sqrt{2} \times (5 \times \sqrt{2}) = (\sqrt{2} \times 5) \times \sqrt{2} = (5 \times \sqrt{2}) \times \sqrt{2}$   
 $= 5 \times \sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$

**One is the multiplicative neutral** If  $a \in \mathbb{R}$ , then  $a \cdot 1 = 1 \cdot a = a$

**for example** :  $2\sqrt{5} \times 1 = 1 \times 2\sqrt{5} = 2\sqrt{5}$

**Every real number  $\neq 0$  has a multiplicative inverse**

If  $a \neq 0$

It exist an real number  $\frac{1}{a}$  such that

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \text{ (1 is the neutral element of multiplication)}$$

**for example**: the multiplicative inverse for  $\frac{\sqrt{3}}{2}$  is  $\frac{2}{\sqrt{3}}$

$$\text{where } \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$$

**Remark**:  $\frac{a}{b} = a \times \frac{1}{b}, b \neq 0$

i.e.  $\frac{a}{b} = a \times$  the multiplicative inverse of  $b$ .

**Discuss with your teacher: is the division operation commutative in  $\mathbb{R}$ ? Is the division operation associative in  $\mathbb{R}$ ?**





## Examples



**Write** down each of the following numbers  $\frac{6}{\sqrt{2}}$ ,  $-\frac{5}{\sqrt{3}}$ ,  $\frac{15}{2\sqrt{5}}$  where the denominator is an integer.

## Solution

Note that the multiplicative neutral is 1 and it can be written in the form  $\frac{\sqrt{2}}{\sqrt{2}}$  or  $\frac{\sqrt{3}}{\sqrt{3}}$  or  $\frac{\sqrt{5}}{\sqrt{5}}$  or ...

$$\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = \frac{3\sqrt{2}}{1} = 3\sqrt{2}$$

$$-\frac{5}{\sqrt{3}} = -\frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{5\sqrt{3}}{3}$$

$$\frac{15}{2\sqrt{5}} = \frac{15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{2 \times 5} = \frac{3\sqrt{5}}{2}$$



## Practice

1



**Complete** the following to have a true sentence:

A  $\sqrt{2} + \sqrt{2} + \sqrt{2} = \quad \times \sqrt{2} =$

B  $3 \times \sqrt{5} = \sqrt{5} \times$

C  $\sqrt{7} \times \sqrt{7} =$

D  $2\sqrt{5} \times 3\sqrt{5} =$

E The multiplicative neutral in  $\mathbb{R}$  is the number

F The multiplicative inverse for  $\frac{3}{\sqrt{2}}$  is

2



**Write** each of the following numbers such that the denominator is an integer:

A  $\frac{15}{\sqrt{6}}$

B  $\frac{8}{3\sqrt{2}}$

C  $-\frac{6}{\sqrt{3}}$

D  $\frac{25}{2\sqrt{10}}$

Distribution of multiplication  
on addition

For any three real numbers  $a, b, c$ .

$$a \times (b + c) = (a \times b) + (a \times c) = a b + a c$$

$$(a + b) \times c = (a \times c) + (b \times c) = a c + b c$$





**Examples**

1 Simplify the following to the simplest form .

A  $2\sqrt{5} (3 + \sqrt{5})$

B  $(\sqrt{2} + 5) (3 + \sqrt{2})$

C  $(2 - 3\sqrt{5})^2$

**Solution**

$$\begin{aligned} \text{A } 2\sqrt{5} (3 + \sqrt{5}) &= 2\sqrt{5} \times 3 + 2\sqrt{5} \times \sqrt{5} \\ &= 2 \times 3 \times \sqrt{5} + 2 \times 5 = 6\sqrt{5} + 10 \end{aligned}$$

$$\begin{aligned} \text{B } (\sqrt{2} + 5) (3 + \sqrt{2}) &= \sqrt{2} (3 + \sqrt{2}) + 5(3 + \sqrt{2}) \\ &= \sqrt{2} \times 3 + \sqrt{2} \times \sqrt{2} + 5 \times 3 + 5 \times \sqrt{2} \\ &= 3\sqrt{2} + 2 + 15 + 5\sqrt{2} \\ &= 3\sqrt{2} + 17 + 5\sqrt{2} = 8\sqrt{2} + 17 \end{aligned}$$

$$\begin{aligned} \text{C } (2 - 3\sqrt{5})^2 &= (2)^2 + 2 \times 2 \times -3\sqrt{5} + (-3\sqrt{5})^2 \\ &= 4 - 12\sqrt{5} + 9 \times 5 \\ &= 49 - 12\sqrt{5} \end{aligned}$$

2 Give an estimation to the result of  $(3 + \sqrt{5}) \times (1 + \sqrt{8})$ , then check your answer using the calculator.

**Solution**

**First:** The estimate of  $\sqrt{5}$  is 2  $\therefore (3 + \sqrt{5})$  the estimate of  $3 + 2 = 5$

the estimate of  $\sqrt{8}$  is 3  $\therefore (1 + \sqrt{8})$  the estimate of  $1 + 3 = 4$

$\therefore (3 + \sqrt{5}) (1 + \sqrt{8})$  the estimate of  $5 \times 4 = 20$

**Second:** when we use the calculator to find  $(3 + \sqrt{5}) \times (1 + \sqrt{8})$

We find that the result is 20.0459

Therefore, the estimate is reasonable.



## Exercises (1-7)

1  **Choose** the correct sentence:

- A  $2\sqrt{3} + 3\sqrt{3} = \dots\dots\dots$  ( $5\sqrt{6}$  or  $5\sqrt{3}$  or  $6\sqrt{3}$  or  $5\sqrt[3]{3}$ )  
 B  $\sqrt[3]{5} + \sqrt[3]{5} = \dots\dots\dots$  ( $\sqrt[3]{10}$  or 5 or  $2\sqrt[3]{5}$  or  $5\sqrt[3]{5}$ )  
 C  $5 + 7\sqrt{2} - 4 + \sqrt{2} = \dots\dots\dots$  ( $15$  or  $1 + 7\sqrt{2}$  or  $1 + 8\sqrt{2}$  or  $1 + 6\sqrt{2}$ )  
 D  $-2\sqrt{3} \times \sqrt{3} = \dots\dots\dots$  ( $-6$  or  $-2\sqrt{3}$  or  $2\sqrt{3}$  or  $6$ )  
 E  $\frac{6}{\sqrt{3}} = \dots\dots\dots$  ( $\sqrt{2}$  or 2 or  $2\sqrt{3}$  or  $6\sqrt{3}$ )  
 F  $(2\sqrt[3]{5})^3 = \dots\dots\dots$  ( $10$  or  $20$  or  $4\sqrt[3]{5}$  or  $40$ )

2 **Simplify to the simplest form:**

- A  $\sqrt{2}(5 + \sqrt{2})$  B  $\sqrt{7}(\sqrt{7} + 2)$   
 C  $-\sqrt{3}(-5 - \sqrt{3})$  E  $(\sqrt{2} + 1)(\sqrt{2} - 1)$

3  **Write** each of the following numbers where the denominator is an integer:

- A  $\frac{10}{\sqrt{5}}$  B  $\frac{8}{\sqrt{6}}$   
 C  $\frac{6}{2\sqrt{3}}$  D  $\frac{\sqrt{2} + 3}{\sqrt{2}}$

4 **Simplify to the simplest form:**

- A  $2\sqrt{3} + 5 + \sqrt{3} - 6$  B  $2\sqrt{7} - 3\sqrt{2} + \sqrt{7} + 5\sqrt{7}$   
 C  $(\sqrt{3} + 2)(\sqrt{3} - 1)$  D  $\sqrt{5}(3 - \sqrt{5}) - 2(1 + \sqrt{5})$

5 If  $a = \sqrt{3} + 2$ ,  $b = \sqrt{3} - 2$  Find the value of each of the following :

- A  $a + b$  B  $a - b$  C  $a b$

6 If  $X = \sqrt{15} + 2$ ,  $Y = 4 - \sqrt{25}$  estimate the value :

- A  $X, Y$  B  $X \times Y$  C  $X + Y$

Check the reasonability of each value using your calculator.





Operations on the square  
roots

## Think and Discuss

If  $a$ ,  $b$  are two non-negative real numbers, then

First:  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

For example :  $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

$$\sqrt{2} \times \sqrt{10} = \sqrt{2 \times 10} = \sqrt{20}$$

$$\sqrt{15} \times \sqrt{5} = \sqrt{15 \times 5} = \sqrt{75}$$

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

For example :  $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

$$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$$

Second:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } b \neq 0$$

For example :  $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{1}{3} \sqrt{5}$

$$\sqrt{\frac{16}{3}} = \frac{\sqrt{16}}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

Third:

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} \quad b \neq 0$$

For example :  $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$

$$\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

## You will learn how

- ✎ To conduct operations on the square roots.
- ✎ To multiply two conjugates.

## key terms

- ✎ Square root.
- ✎ Two conjugates numbers .





## Examples

- 1 Simplify to the simplest form  $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

**Solution**

$$\begin{aligned}\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}} &= \sqrt{16 \times 2} - \sqrt{36 \times 2} + 6 \times \frac{\sqrt{1}}{\sqrt{2}} \\ &= \sqrt{16} \times \sqrt{2} - \sqrt{36} \times \sqrt{2} + 6 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = \sqrt{2}\end{aligned}$$

- 2 If  $x = 2\sqrt{5} - 1$ ,  $Y = 2 + \sqrt{5}$  find the value of  $x^2 + y^2$

**Solution**

$$\begin{aligned}x^2 &= (2\sqrt{5} - 1)^2 = (2\sqrt{5})^2 - 4\sqrt{5} + 1 \\ &= 4 \times 5 - 4\sqrt{5} + 1 = 21 - 4\sqrt{5} \\ y^2 &= (2 + \sqrt{5})^2 = 4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5} \\ x^2 + y^2 &= 21 - 4\sqrt{5} + 9 + 4\sqrt{5} = 30\end{aligned}$$



## Practice

- 1 put each of the following in the form of  $a\sqrt{b}$  where  $a$  and  $b$  are integers,  $b$  is the least possible value:

A  $\sqrt{28}$

B  $\sqrt{75}$

C  $\sqrt{54}$

D  $\sqrt{1000}$

E  $2\sqrt{72}$

F  $\frac{1}{3}\sqrt{162}$

- 2 Simplify to the simplest form:

A  $2\sqrt{18} \times 3\sqrt{2}$

B  $\sqrt{5} \times 2\sqrt{10}$

C  $3\sqrt{7} \times 2\sqrt{28}$

D  $\sqrt{50} + \sqrt{8}$

E  $\sqrt{20} - \sqrt{45}$

F  $\sqrt{27} + 5\sqrt{18} - \sqrt{300}$

- 3 Find the value of  $X + Y$ ,  $X \times Y$  in each of the following cases:

A  $x = 3 + \sqrt{5}$ ,  $y = 1 - \sqrt{5}$

B  $x = \sqrt{3} - \sqrt{2}$ ,  $y = \sqrt{3} + \sqrt{2}$

C  $x = 5 - 3\sqrt{2}$ ,  $y = 5 - 3\sqrt{2}$





### The two conjugate numbers

If  $a$  and  $b$  are two positive rational numbers.

Then each of the two number  $(\sqrt{a} + \sqrt{b})$ ,  $(\sqrt{a} - \sqrt{b})$  is a conjugate to the other one.

then, their sum is  $= 2\sqrt{a}$  twice the first term

and their product is  $= (\sqrt{a} + \sqrt{b}) (\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

= The square of the first term - The square of the second term

The product of two conjugates is always a rational number

If we have a real number whose denominator is written in the form  $(\sqrt{a} \pm \sqrt{b})$ , we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.



#### Practice



#### Complete

- A  $\sqrt{5} + \sqrt{2}$  their conjugate ( ..... ) and their product is ( ..... )
- B  $5 - \sqrt{3}$  their conjugate ( ..... ) and their product is ( ..... )
- C  $2\sqrt{3} + \sqrt{2}$  their conjugate ( ..... ) and their product is ( ..... )



#### Examples



1 Given  $x = \frac{8}{\sqrt{5} - \sqrt{3}}$ ,  $y = \frac{2 - \sqrt{3}}{2 + \sqrt{3}}$



Write both of  $X$  and  $Y$  where the denominator is a rational number, then find  $X + Y$

#### Solution

$$\begin{aligned} x &= \frac{8}{\sqrt{5} - \sqrt{3}} = \frac{8}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \\ &= \frac{8(\sqrt{5} + \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} = \frac{8(\sqrt{5} + \sqrt{3})}{5 - 3} = 4(\sqrt{5} + \sqrt{3}) \end{aligned}$$



$$y = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$= \frac{(2 - \sqrt{3})^2}{4 - 3} = \frac{4 - 4\sqrt{3} + 3}{1} = 7 - 4\sqrt{3}$$

$$x + y = 4\sqrt{5} + 4\sqrt{3} + 7 - 4\sqrt{3} = 4\sqrt{5} + 7$$

2 Given  $x = \frac{4}{\sqrt{7} - \sqrt{3}}$ ,  $y = \sqrt{7} - \sqrt{3}$ ,



**prove that**  $x$  and  $y$  are conjugates, then find the values of:

$x^2 - 2xy + y^2$ ,  $(x - y)^2$ . What do you observe?

**Solution**

$$x = \frac{4}{\sqrt{7} - \sqrt{3}} \times \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} + \sqrt{3}} = \frac{4(\sqrt{7} + \sqrt{3})}{7 - 3} = \sqrt{7} + \sqrt{3}$$

$$y = \sqrt{7} - \sqrt{3} \therefore x, y \text{ (two conjugate numbers)}$$

$$x^2 - 2xy + y^2 = (\sqrt{7} + \sqrt{3})^2 - 2(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}) + (\sqrt{7} - \sqrt{3})^2$$

$$= (7 + 2\sqrt{21} + 3) - 2(7 - 3) + (7 - 2\sqrt{21} + 3)$$

$$= 10 + 2\sqrt{21} - 8 + 10 - 2\sqrt{21}$$

$$= 12$$

$$(x - y)^2 = [(\sqrt{7} + \sqrt{3}) - (\sqrt{7} - \sqrt{3})]^2$$

$$= [\sqrt{7} + \sqrt{3} - \sqrt{7} + \sqrt{3}]^2 = (2\sqrt{3})^2$$

$$= 4 \times 3 = 12$$

**Remark :**

$$x^2 - 2xy + y^2 = (x - y)^2$$



**Practice**

In the previous example, find the value of each of the following:

A  $(x + y)$

B  $(x - y)$

C  $(x + y)(x - y)$


D  $x^2 - y^2$

What do you observe?







## Exercises (1-8)

1  **Choose** the correct answer :

- A  $\sqrt{50} - \sqrt{18} - \sqrt{2} = \dots\dots\dots$  ( $\sqrt{30}$  or  $\sqrt{2}$  or 2 or  $2\sqrt{2}$ )  
 B  $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) = \dots\dots\dots$  (2 or 12 or  $2\sqrt{7}$  or  $-2\sqrt{5}$ )  
 C  $(\sqrt{8} + \sqrt{2})^2 = \dots\dots\dots$  ( $\sqrt{10}$  or 10 or 18 or  $\sqrt{18}$ )  
 D The multiplicative inverse of  $\frac{\sqrt{3}}{6}$  is  $\dots\dots\dots$  ( $-\frac{\sqrt{3}}{6}$  or  $6\sqrt{3}$  or  $2\sqrt{3}$  or  $-2\sqrt{3}$ )  
 E The next number in the pattern:  $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$  is  $\dots\dots\dots$  ( $\sqrt{50}$  or  $\sqrt{75}$  or  $\sqrt{60}$  or  $\sqrt{90}$ )

2  **Complete** the following to get a correct sentence:

- A If  $X = 3 + \sqrt{2}$  then the conjugate is  $\dots\dots\dots$  and the product of the number by its conjugate is  $\dots\dots\dots$   
 B The multiplicative inverse for  $(\sqrt{3} + \sqrt{2})$  in its simplest form is  $\dots\dots\dots$   
 C  **Think** If  $x^2 = 5$ , then  $(x + \sqrt{5})^2 = \dots\dots\dots$  or  $\dots\dots\dots$   
 D  **Think** If  $\frac{1}{x} = \sqrt{5} - 2$ , then the value of X in its simplest form is  $\dots\dots\dots$   
 E  $3\sqrt{2} + \sqrt{8} - \sqrt{18} = \dots\dots\dots$

3 Simplify to the simplest form  $2\sqrt{5} + 6\sqrt{\frac{1}{3}} - \sqrt{12} - 5\sqrt{\frac{1}{5}}$

4 If  $x = \frac{4}{\sqrt{7} - \sqrt{3}}$ ,  $y = \frac{4}{\sqrt{7} - \sqrt{3}}$ . Find the value of  $x^2 y^2$

5 If  $A = \sqrt{3} + \sqrt{2}$ ,  $B = \frac{1}{\sqrt{3} + \sqrt{2}}$ . Find the value of  $A^2 - B^2$  in its simplest form.

6 If  $x = \sqrt{5} + \sqrt{2}$ ,  $y = \sqrt{5} - \sqrt{2}$ .

Find the value of  $\frac{x+y}{xy-1}$  in its simplest form

7 If  $x = \sqrt{7} + \sqrt{5}$ ,  $y = \frac{2}{x}$

find the value of  $\frac{x+y}{xy}$  in its simplest form



# Unit One

## Lesson Nine

## Operations on the cube roots

### Think and Discuss

#### You will learn how

- To carry operations on the cube roots.

#### key terms

- Cube root.

For any two real numbers  $a, b$  :

1

$$\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{a \times b}$$

For example :  $\sqrt[3]{5} \times \sqrt[3]{2} = \sqrt[3]{5 \times 2} = \sqrt[3]{10}$

$$\sqrt[3]{3} \times \sqrt[3]{-4} = \sqrt[3]{3 \times -4} = \sqrt[3]{-12}$$

2

$$\sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}$$

For example :  $\sqrt[3]{40} = \sqrt[3]{8 \times 5} = \sqrt[3]{8} \times \sqrt[3]{5} = 2\sqrt[3]{5}$

$$\sqrt[3]{-128} = \sqrt[3]{-64 \times 2} = \sqrt[3]{-64} \times \sqrt[3]{2} = -4\sqrt[3]{2}$$

3

$$\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}} \text{ where } b \neq 0, a, b \in \mathbb{R}$$

For example :  $\frac{\sqrt[3]{12}}{\sqrt[3]{3}} = \sqrt[3]{\frac{12}{3}} = \sqrt[3]{4}$

4

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}} \text{ where } b \neq 0, a, b \in \mathbb{R}$$

For example :  $\sqrt[3]{\frac{3}{2}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2}}$



**Think :** If we multiply both the numerator and the denominator by  $\sqrt[3]{4}$ , then find the product in its simplest form







### Examples :

Simplify to the simplest form:

A  $\sqrt[3]{54} + 8\sqrt[3]{\frac{-1}{4}} + 5\sqrt[3]{16}$

B  $\sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}}$

### Solution

$$\begin{aligned}
 \text{A } \sqrt[3]{54} + 8\sqrt[3]{\frac{-1}{4}} + 5\sqrt[3]{16} &= \sqrt[3]{27 \times 2} + 8\sqrt[3]{\frac{-1}{4} \times \frac{2}{2}} + 5\sqrt[3]{8 \times 2} \\
 &= \sqrt[3]{27} \times \sqrt[3]{2} + 8\frac{\sqrt[3]{-2}}{\sqrt[3]{8}} + 5 \times \sqrt[3]{8} \times \sqrt[3]{2} \\
 &= 3\sqrt[3]{2} + \frac{8 \times (-\sqrt[3]{2})}{2} + 5 \times 2 \times \sqrt[3]{2} \\
 &= 3\sqrt[3]{2} - 4\sqrt[3]{2} + 10\sqrt[3]{2} = 9\sqrt[3]{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{B } \sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}} &= \sqrt[3]{24} - 6\sqrt[3]{\frac{125}{9}} = \sqrt[3]{8 \times 3} - 6 \times \sqrt[3]{\frac{125}{9} \times \frac{3}{3}} \\
 &= \sqrt[3]{8} \times \sqrt[3]{3} - 6 \times \frac{5\sqrt[3]{3}}{3} = 2\sqrt[3]{3} - 2\sqrt[3]{10} = -8\sqrt[3]{3}
 \end{aligned}$$

## Exercises (1-9)

- 1 Put each of the following in the form  $a\sqrt[3]{b}$  where a and b are two integers, b is the least positive possible value:

A  $\sqrt[3]{54}$

B  $\sqrt[3]{-1000}$

C  $\sqrt[3]{128}$

D  $\sqrt[3]{-2160}$

E  $\sqrt[3]{1715}$

F  $\sqrt[3]{686}$

- 2 Find the result for each of the following in the simplest form:

A  $\sqrt[3]{125} - \sqrt[3]{24}$

B  $\sqrt[3]{250} - \sqrt[3]{128}$

C  $\sqrt[3]{\frac{2}{5}} \times \sqrt[3]{\frac{4}{25}}$

D  $\sqrt[3]{\frac{3}{4}} + \sqrt[3]{\frac{2}{9}}$

E  $\frac{1}{2}\sqrt[3]{56} - \sqrt[3]{\frac{7}{27}}$

F  $\frac{1}{2}\sqrt[3]{10} \times 6\sqrt[3]{100}$

- 3 If  $a = \sqrt[3]{5} + 1$ ,  $b = \sqrt[3]{5} - 1$  find the value of each of the following

A  $(a - b)^5$

B  $(a + b)^3$

- 4 Prove that:

A  $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = \text{zero}$

B  $\sqrt[3]{54} \times \sqrt[3]{16} \div (\sqrt[3]{4} \times 6) = 1$



# Unit One

## Lesson Ten

### Applications on the real numbers

#### Think and Discuss

#### You will learn how

- To solve applications on square and cube roots.

#### key terms

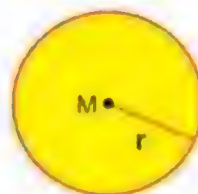
- Circle
- Cuboid
- Cube
- Right circular cylinder
- Sphere

#### The circle:

Circumference of a circle =  $2\pi r$  length unit

area of a circle =  $\pi r^2$  square unit

where  $r$  is the length of the radius in a circle,  $\pi$  is the (approximate ratio).



#### Examples



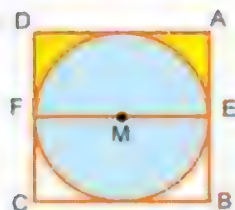
- 1 Find the circumference of a circle whose area is  $38.5 \text{ cm}^2$  ( $\pi = \frac{22}{7}$ )

#### Solution

The area of the circle =  $\pi r^2$

$$38.5 = \frac{22}{7} r^2 \quad \therefore r^2 = \frac{38.5 \times 7}{22} = \frac{49}{4}$$

$$\therefore r = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3.5 \text{ cm}$$



- 2 In the opposite figure, the circle M is inside the square ABCD. If the area of the yellow sector is  $10 \frac{5}{7} \text{ cm}^2$ , find the perimeter of the sector ( $\pi = \frac{22}{7}$ )

#### Solution

We suppose that the length of the radius in a Circle =  $r$

$\therefore$  The side length of the square =  $2r$



The area of the yellow color = the area of the rectangle AEFD - the area of semi circle

$$10 \frac{5}{7} = r \times 2r - \frac{1}{2} \times \frac{22}{7} r^2$$

$$\frac{75}{7} = 2r^2 - \frac{11}{7} r^2 = \frac{3}{7} r^2$$

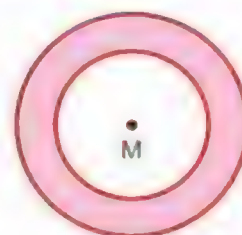
$$\therefore r^2 = 25 \quad \therefore r = 5 \text{ cm}$$

The perimeter of the yellow sectors = (AE + AD + DF) +  $\frac{1}{2}$  the circumference of the circle  
 $= (5 + 10 + 5) + \frac{1}{2} \times 2 \times \frac{22}{7} \times 5 = 35 \frac{5}{7} \text{ cm}$



### Practice

1. A circle whose area is  $64\pi \text{ cm}^2$ . Find the length of its radius, then find its circumference approximating it to the nearest integer ( $\pi = 3.14$ ).
2. In the figure opposite:  $\overline{AB}$  is the diameter of a semi circle. If the area of that region is  $12.32 \text{ cm}^2$ . Find the circumference of that figure.
3. In the opposite figure: there are two circles have the same center "concentric" of center M. If the lengths of their radii are 3cm and 5cm. Find the area and the circumference of the colored region in the terms of  $\pi$ .



### The cuboid

It is a body whose six faces are of a rectangular shape such that every two opposite faces are congruent.:

If the lengths of its edges were  $x, y, z$ , then:

The lateral area = the perimeter of the base  $\times$  the height

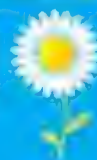
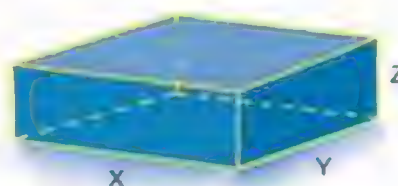
The lateral area =  $2(x + y) \times z$  square unit

The lateral area = the lateral area +  $2 \times$  the area of the base

The total area =  $2(xy + yz + Xz)$  square unit

The volume of the cuboid = the area of the base  $\times$  the height

The volume of the cuboid =  $x \times y \times z$  cubic unit





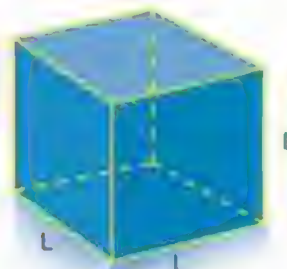
### A special case : the cube

It is a cuboid whose edges are equal in length. If the length of one edge =  $L$  length unit, then:

The area of each face =  $L^2$  square unit

The lateral area of each face =  $4L^2$  square unit

The total area =  $6L^2$  square unit, the volume of the cube =  $L^3$  cubic unit



### Examples



**Find** the total area of a cube whose volume is  $125 \text{ cm}^3$

#### Solution

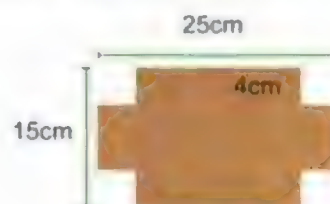
The volume of the cube =  $L^3$   $\therefore 125 = L^3$   $\therefore L = \sqrt[3]{125} = 5 \text{ cm}$

The total area =  $6L^2 = 6 \times (5)^2 = 150 \text{ cm}^2$



### Practice

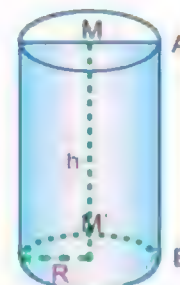
- 1 Find the total area of a cuboid whose volume is  $720 \text{ cm}^3$  and height  $5 \text{ cm}$  with a squared shape base.
- 2 Which is more in volume: A cube of  $294 \text{ cm}^2$  area or a cuboid with the following dimensions:  $7\sqrt{2}$ ,  $5\sqrt{2}$ ,  $5 \text{ cm}$ .
- 3 A rectangular hard piece of paper has a length of  $25 \text{ cm}$  and a width of  $15 \text{ cm}$ . A square whose side =  $4 \text{ cm}$  was cut from each of its four corners. Then, the projected parts were folded to form a shape of a cuboid. Find the volume and the total area of that cuboid.



### The right circular cylinder :

It is a body that has two parallel congruent bases each is a circular shaped surface, while its lateral surface is a curved surface called cylindrical surface.

- If  $M$ ,  $M'$  are the bases of the cylinder, then  $MM'$  is the height of cylinder.





**Let's think** If  $A \in$  the circle  $M$ ,  $B \in$  the circle  $M'$ ,

$$\overline{AB} \parallel \overline{MM'}$$

- Then, if we cut the lateral cylindrical surface at  $AB$  and we stretch That surface, we get the surface of the rectangle  $A B B' A'$

Then,  $AB$  = height of cylinder,  $A A'$  = the perimeter of the base of the cylinder.



The area of the rectangle  $A B B' A'$  = the lateral area of the cylinder

The lateral area of the cylinder = the perimeter of the base  $\times$  height =  $2\pi r h$  (square unit)

the total area of the cylinder = area of lateral surface + sum of the areas of the two bases

$$= 2\pi r h + 2\pi r^2 \quad (\text{square unit})$$

the volume of the cylinder = base area  $\times$  height =  $\pi r^2 h$  (cubic unit)



### Example

A piece of paper has shape of a rectangle  $ABCD$  in which  $AB = 10\text{cm}$ ,  $BC = 44\text{cm}$ . It was folded to form a right circular cylinder such that  $\overline{AB}$  is congruent to  $\overline{DC}$ . Find the volume of the resulted cylinder. ( $\pi = \frac{22}{7}$ ).

### Solution

The perimeter of the cylinder base = 44 cm.

$$2\pi r = 44$$

$$2 \times \frac{22}{7} r = 44$$

$$\therefore r = 7\text{cm}$$

The volume of the cylinder =  $\pi r^2 h$

$$= \frac{22}{7} \times (7)^2 \times 10$$

$$= 1540 \text{ cm}^3$$



### Practice

- Find the volume and the total area of a right circular cylinder in which the length of base radius = 14 cm and the height is 20 cm .



- 2 Find the total area of a right circular cylinder of volume  $7536 \text{ cm}^3$  and height  $24 \text{ cm}$  ( $\pi = 3.14$ )
- 3 Which is more in volume: a right circular cylinder of radius  $7 \text{ cm}$  and height  $10 \text{ cm}$  or a cube whose edge length is equal to  $11 \text{ cm}$

### The sphere:

It is a body of curved surface in which the points have the same distance ( $r$ ) from a constant point inside it (the center of the sphere)..

If the sphere is cut by a plane passing by its center, then the resulted section is a circle whose center is the center of a sphere where its radius is the radius of a sphere ( $r$ ).



Volume of the sphere =  $\frac{4}{3} \pi r^3$       cubic units.

area of the sphere =  $4 \pi r^2$       square units.



### Examples

The volume of the sphere is  $562.5 \pi \text{ cm}^3$ . Find its surface area.

#### Solution

$$\text{the volume of sphere} = \frac{4}{3} \pi r^3$$

$$562.5 \pi = \frac{4}{3} \times \pi r^3$$

$$\therefore r^3 = 562.5 \times \frac{3}{4} = 421.875$$

$$r = \sqrt[3]{421.875} = 7.5 \text{ cm}$$

$$\text{the surface area of sphere} = 4 \pi r^2 = 4 \times \pi (7.5)^2 = 225 \pi \text{ cm}^2$$




### Practice

Find the volume and the surface area of a sphere whose diameter is  $4.2 \text{ cm}$  ( $\pi = \frac{22}{7}$ )





### Exercises (1-10)

 **Choose** the correct answer

- A** The lateral area of a right circular cylinder whose base diameter length  $b$  and height  $h$  is ....  
( $\pi b^2h$ ,  $\pi bh$ ,  $2\pi bh$ ,  $\pi bh^2$ ).
- B** The volume of a sphere whose diameter length is  $6\text{cm} = \dots \text{cm}^3$   
( $288$ ,  $12\pi$ ,  $36\pi$ ,  $288\pi$ )
- C** The edge length of a cube whose volume is  $2\sqrt{2} \text{cm}^3 = \dots \text{cm}$ .  
( $\sqrt{2}$ ,  $2$ ,  $8$ ,  $1.5$ )
- D** The radius length of a right circular cylinder whose volume is  $40\pi \text{cm}^3$  and height  $10\text{cm} = \dots \text{cm}$   
( $5$ ,  $3$ ,  $2$ ,  $1$ ).
- E** The volume of a cuboid whose dimensions are  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{6} \text{cm}$  is .....  
( $6$ ,  $36$ ,  $6\sqrt{6}$ ,  $18\sqrt{2}$ ).

**2**  **Complete**

- A** The radius length of a sphere whose volume is  $\frac{9}{2} \pi \text{ cm}^3$  is ..... cm.
- B** A right circular cylinder whose base radius length  $r$  and height  $h$ , then its lateral area is .... and its volume is ....
- C** The total area of a cube of edge length = 4 cm is ....  $\text{cm}^2$ .
- D** The lateral area of the cuboid = .....

**3** A sphere with volume  $36\pi$  cm<sup>3</sup> is placed inside a cube, if the sphere touches the cube's six faces. Find

- A** the radius of the sphere.      **B** the volume of the cube.

**4** A metal sphere with diameter 6 cm has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height.

**5** Find the height of a right circular cylinder whose height is equal to its base radius and its volume is  $72\pi \text{ cm}^3$ .

**6** A hollow metal sphere with internal radius 2.1 cm and external radius 3.5 cm. Find its mass to the nearest gram given that the mass of a cubic centimeter of such a metal is 20 gm ( $\pi \frac{22}{7}$ )



# Unit One

## Lesson Eleven

### Solving Equations and Inequalities of first degree in one variable in R

#### Think and Discuss

#### You will learn how

- ☞ To solve equation of first degree in one variable in R.
- ☞ To solve inequalities of first degree in one variable

#### key terms

- ☞ -equation
- ☞ degree of an equation.
- ☞ Inequality
- ☞ degree of an inequality
- ☞ Solution of an equation
- ☞ Solution of an inequality

First: Solving Equations of first degree in one variable in R

**We know that:** The equation  $3X - 2 = 4$  is called an equation of first degree where the exponent of the (unknown) variable X is 1. To solve that equation in R

$$3x - 2 = 4 \quad \text{By adding 2 to the sides of the equation}$$

$$3x = 6 \quad (\text{we can multiply by the multiplicative inverse of the coefficient of X})$$

$$\frac{1}{3} \times 3x = \frac{1}{3} \times 6$$

$$\therefore x = 2$$

i.e the solution set { 2 }

This solution can be graphed on the number line as shown in the figure opposite .



#### Examples

1



**Find** the solution set of the equation  $\sqrt{3}x - 1 = 2$ , in R, then graph the solution on the number line.

#### Solution

$$\sqrt{3}x - 1 = 2 \quad \therefore \sqrt{3}x = 3$$

$$\therefore x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \quad \therefore x = \sqrt{3} \in \mathbb{R}$$


The solution set is  $\{\sqrt{3}\}$



This solution can be graphed on the number line as shown in the figure opposite .





- 2  **Find** The solution set for the equation  $x + \sqrt{2} = 1$ , in  $\mathbb{R}$ , then graph the solution on the number line.

**Solution**


$$x + \sqrt{2} = 1 \quad \therefore x = 1 - \sqrt{2} \in \mathbb{R}$$



This can be graphed on the number as shown in the figure opposite .



### Practice

- 1  **Find** the solution set for the following equations in  $\mathbb{R}$ , then graph the solution on the line number.

A  $5x + 6 = 1$

B  $2x + 4 = 3$

C  $2x - 3 = 4$

D  $x + 5 = 0$

E  $\sqrt{2}x - 1 = 1$

F  $x - 1 = \sqrt{5}$

**Second : Solving inequalities of the first degree in one variable in  $\mathbb{R}$ , graphing the solution on the number line.**


The following properties are used to solve the inequality in  $\mathbb{R}$ . The solution set is written in the form of an interval

**If  $A, B, C$  were real number where  $A < B$ , then:**

- 1  $A + C < B + C$ . addition property.
- 2 If  $C > 0$  then  $A \times C < B \times C$ . property of multiplication by a positive real number
- 3 If  $C < 0$  then  $A \times C > B \times C$ . property of multiplication by negative real number.



### Examples

- 1  **Find** the solution set for the inequality  $2x - 1 \geq 5$  in  $\mathbb{R}$  and represent the solution set graphically.

**Solution**

By adding 1 to the sides of the inequality it becomes  $2x \geq 6$


by multiplying the side of the inequality by  $(\frac{1}{2} > 0)$   $x \geq 3$

$\therefore$  The solution set in  $\mathbb{R}$  is  $[3, \infty[$

and it is graphed by green color ray on the number line.





- 2  **Find** the solution set for the inequality  $5 - 3x > 11$ , in  $\mathbb{R}$ , then represent the solution graphically.


**Solution**

By adding  $(-5)$  to the sides of the inequality then  $-3x > 6$   
by multiplying the sides of the inequality by  $(-\frac{1}{3} < 0)$  we get :  
 $\therefore x < -2$

i.e., the solution set in  $\mathbb{R}$  is est  $]-\infty, -2[$

and it is represented by the green color ray on the number line.



- 3  **Find** the solution set for the inequality  $-3 \leq 2x - 1 < 5$  in  $\mathbb{R}$  and represent the solution graphically.


**Solution**

by adding  $(1)$  to the sides of the inequality  $-3 + 1 \leq 2x - 1 + 1 < 5 + 1$   
Namely,  $-2 \leq 2x < 6$ , and by multiplying the sides of the inequality by  $(\frac{1}{2} > 0)$   $-1 \leq x < 3$   
 $\therefore$  the solution set in  $\mathbb{R}$  is  $[-1, 3[$  and it is graphed on the number line by the green color.




- in example 3 What is the solution set for the inequality in  $\mathbb{N}$ ?  
What is the solution set for the inequality in  $\mathbb{Z}$ ?


## Exercises (1-11)

- 1  **Complete** the following to have a true sentence where  $X \in \mathbb{R}$
- A If  $5x < 15$ , then  $x$  .....
  - B If  $x - 3 > 4$ , then  $x$  .....
  - C If  $-2x < 3$ , then  $x$  .....
  - D If  $1 - x > 4$ , then  $x$  .....
  - E If  $\sqrt{2}x \leq 4$ , then  $x$  .....




- 2  **Find** The solution set for each of the following inequalities in  $\mathbb{R}$  in the form of intervals, then graph the solution on the number line.:

- A  $3x - 1 < 5$  B  $2x + 5 > 3$   
 C  $2x + 3 < 1$  D  $5 - x > 3$   
 E  $1 - 5x < 6$  F  $\frac{1}{2}x + 1 < 2$

- 3  **Find** the solution set for each of the following inequalities in  $\mathbb{R}$  in the form of an interval, then represent the solution on the number line.

- A  $-1 < 2x + 1 < 5$  B  $-5 < 2x - 3 < 1$   
 C  $-3 < 4x - 7 < 5$  D  $4 < 3x + 4 < 7$   
 E  $1 < 5 - x < 3$  F  $1 < 3 - 2x < 5$

- 4  **Find** the solution set for each of the following inequalities in  $\mathbb{R}$ , write it in the form of an interval, then represent the solution on the number line.:

- A  $-3 \leq -x < 3$  B  $|-3| < 2x - 1 < 5$   
 C  $\sqrt{-8} \leq x + 1 \leq \sqrt{9}$  D  $5 < 3 - x \leq 3^2$

## General Exercises

- 1  **Complete** the following to have a true sentence:

- A  $\sqrt{9} + \sqrt[3]{-8} = \dots\dots\dots$   
 B A vessel that has the shape of a cub whose capacity is 8 liters, the length of the edge of its interior face is  $\dots\dots\dots$  cm.  
 C The solution set for the equation  $x^2 + 9 = 0$  in  $\mathbb{R}$  is  $\dots\dots\dots$   
 D  $(\sqrt{3} + \sqrt{2})^2 + (\sqrt{3} - \sqrt{2})^2 = \dots\dots\dots$   
 E A rectangle of dimensions are  $(\sqrt{5} + 1)$ ,  $(\sqrt{5} - 1)$  cm has an area of  $\dots\dots\dots$  cm<sup>2</sup>.  
 F  $\sqrt[3]{54} - \sqrt[3]{-16} = \sqrt[3]{\dots\dots\dots}$   
 G  $[-1, 5] - ]-1, 5[ = \dots\dots\dots$   
 H The solution set for the equation  $\sqrt{2}x - 1 = 3$  in  $\mathbb{R}$  is  $\dots\dots\dots$   
 I A sphere whose diameter is equal to 6 L (length unit) has a volume of  $\dots\dots\dots$  (cubic unit).  
 J  $|\sqrt[3]{-125}| = \sqrt{\dots\dots\dots}$



2



**Find** the solution set for each of the following inequalities in  $\mathbb{R}$ , in the form of an interval, then represent the solution on the number line.:

A  $5x - 3 < 2x + 9$

B  $3 - 4x \leq x - 2$

C  $x \leq 2x - 1 \leq x + 3$

D  $x - 1 < 3x - 1 \leq x + 1$

E  $4x \leq 5x + 2 < 4x + 3$

F  $5x + 7 > 6x > 5x$

3

If  $x = \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} - \sqrt{5}}$  prove that  $x + \frac{1}{x} = 22$

4



**Find** in the simplest form:  $\sqrt[3]{54} + 4\sqrt[3]{\frac{1}{4}} - \sqrt[3]{-2}$

5

**Find** the total area of a right circular cylinder of volume  $72\pi \text{ cm}^3$ , and height 8 cm.

6



**Find** the following using the number line  $[3, 6] \cap [4, 7]$

7

If  $x = \frac{5\sqrt{2} + 3\sqrt{5}}{\sqrt{5}}$ ,  $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$  Find the value

A  $x^2 + y^2$

B  $xy$

and prove that  $x^2 + y^2 = 38xy$

8

If  $X = \sqrt[3]{5} + 2$ ,  $y = \sqrt[3]{5} - 2$  Find the value of  $(x + y)^3 + (x - y)^3$ .

9

If  $x = \sqrt{5} - \sqrt{3}$ ,  $y = \frac{2}{\sqrt{5} - \sqrt{3}}$ , Find the value of  $(X^2 + 2xy + y^2)$

10

If  $A = \sqrt{3} + \sqrt{2}$ ,  $B = \sqrt{3} - \sqrt{2}$ ,

find the value of  $(A^2 - AB + B^2)$

11

If  $x = \frac{3\sqrt{5} + 5\sqrt{2}}{\sqrt{5}}$ ,  $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$

Prove that  $\frac{x^2 + y^2}{xy} = 38$





## Technology



Find:  $\sqrt[3]{27} + \sqrt{12\frac{1}{4}} + \sqrt[3]{0.125}$

Open microsoft office Excel and record the shown numbers in cells A1, D1 and B1.

A1	B1	C1	D1	E1	F1	G1	H1	I1	J1	K1	L1
27	12.25		0.125		$A1^{(1/3)}$		$B1^{(1/2)}$		$D1^{(1/3)}$		$F2+H2+J2$
					3		3.5		0.5		1

To Find the cube root of cell A1, write the formula  $A1^{(1/3)}$  in the cell F1, then Enter... the result is 3.

To find the square root of cell B1, write the formula  $B1^{(1/2)}$  in the cell H2, then Enter... the result is 3.5.

To find the cube root of cell J1, write the formula  $D1^{(1/3)}$  in the cell J2, then Enter... the result is 0.5.

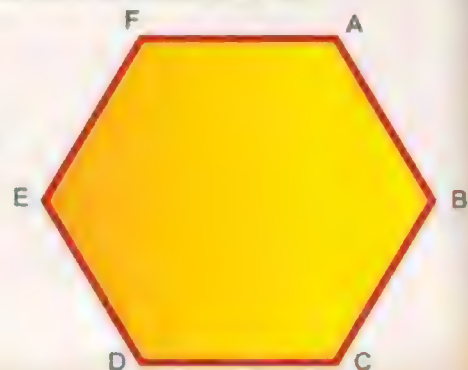
Write down the sum of  $F2+H2+J2$  in cell L2, after you do a click on = then the sum is 1.

## Activity



**Activity** Draw a regular hexagon ABCDEF whose side is equal to 4 cm.

- 1 Find the measure of its interior angle.
- 2 Draw its diagonals  $\overline{AD}$ ,  $\overline{BE}$ ,  $\overline{CF}$  then deduce the length of each diagonal without measuring them by the ruler.
- 3 draw a circle that passes by its vertices.
- 4 Find its area.



## Unit test

1  **Complete** to get a true sentence :

- A  $] -3, 2] \cap \mathbb{R}^+ = \dots\dots\dots$
- B The multiplicative inverse for  $-\frac{\sqrt{2}}{6}$  is  $\dots\dots\dots$
- C  $\sqrt{5}, \sqrt{20}, \sqrt{45}, \sqrt{80}, \dots\dots\dots$  Complete in the same pattern.
- D If  $x = \sqrt[3]{3} + 7, y = \sqrt[3]{3} - 7$  then  $(x + y)^3 = \dots\dots\dots$
- E The circle of 20X cm circumference has an area of  $\dots\dots\dots \pi \text{ cm}^2$

2  **Choose** the correct sentence from Parentheses:

- A a cube of  $64 \text{ cm}^3$  has a lateral area of  $= \dots \text{ cm}^2$  (4 or 8 or 64 or 96)
- B  $\sqrt{12} - \sqrt{3} = \dots\dots\dots$  (3 or  $\sqrt{3}$  or  $2\sqrt{3}$  or  $3\sqrt{3}$ )
- C The multiplicative inverse for  $\frac{-\sqrt{6}}{12}$  is  $(\frac{12}{\sqrt{6}}$  or  $\frac{\sqrt{6}}{12}$  or  $-2\sqrt{6}$  or  $2\sqrt{6})$
- D  $\sqrt[3]{54} + \sqrt[3]{-2} = \dots\dots\dots$  ( $\sqrt[3]{52}$  or  $\sqrt[3]{2}$  or  $2\sqrt[3]{2}$  or  $4\sqrt[3]{2}$ )
- E  $[-3, 4] - \{-3, 5\} = \dots\dots\dots$  ( $] -3, 4[$  or  $] -3, 4]$  or  $] -3, 5[$  or  $[-3, 4[$ )

3 Reduce to the simplest from  $2\sqrt{18} + \sqrt{50} + \frac{1}{3}\sqrt{162}$

4 a lead cuboid in which the lengths of its faces are 77cm, 24cm and 21cm. It was melted to form a sphere. Find the radius of that sphere. ( $\pi = \frac{22}{7}$ )

5 If  $A = \frac{4}{\sqrt{7} - \sqrt{3}}, B = \frac{4}{\sqrt{7} + \sqrt{3}}$  find the value of  $\frac{A-B}{AB}$

6 Using the number line find  $] -1, 3] \cup [0, 5[$  in the form of an interval

7 Find the lateral area for a right circular cylinder of volume  $924 \text{ cm}^3$  and height 6cm ( $\pi = \frac{22}{7}$ ).

8 If  $x = \sqrt{10} + 2, y = \sqrt{26} - 1$ . Give an estimate for the product of  $X \times Y$  and use the calculator to find the difference between your estimate and the exact answer.

9 Find the Solution set in  $\mathbb{R}$  and graph the solution on the number line.

A  $1 < 2x + 3 \leq 9$

B  $x + 2\sqrt{3} = 3$





## UNIT TWO

### 2

## Relation Between Two Variables





# Unit TWO

## Lesson One

# Linear Relation of two variables

### Think and Discuss

#### You will learn how

- The linear Relations of two variables
- To graph the linear relations of two variables

#### key terms

- Variable
- Relation
- Linear equation

A person has some bills of LE 50 and LE 20. He bought an electrical apparatus for LE 390.

**Think:** How many bills of each type does he give to the seller?

**Suppose :** x represents the number of fifties bills, then the value of what he has of these bills is L.E 50x, y represents the number of Twenties bills, then the value of what he has of these bills is L.E 20y.

**Required is to know:** x and y that verify the equation:

$$50x + 20y = 390$$

This relation represents a linear equation in two variables. Dividing both sides over 10 produces the following equivalent equation:

$$5x + 2y = 39$$

$$\therefore y = \frac{39 - 5x}{2}$$

**Note that :** x and y are natural numbers. Therefore, x should be an odd number.

The following table can be created to know the different possibilities of giving bills to the seller: a bill of L.E 50 and 17 bills of L.E 20, or 3 bills of L.E 50 and 12 bills of L.E 20, or 5 bills of L.E 50 and 7 bills of L.E 20, or 7 bills of 50 and 2 bills of L.E 20.

x	y	(x , y)
1	17	(1 , 17)
3	12	(3 , 12)
5	7	(5 , 7)
7	2	(7 , 2)
9	negative	refused





### Practice

- 1 A person has some bills of L.E 5 and some of L.E20. He bought some goods from a shopping center for L.E75. What are the different possibilities of paying this amount in the two types of bills which he has?
- 2 The perimeter of an isosceles triangle is 19cm. What are the different possible lengths of its sides? Side length  $\in \mathbb{Z}_+$

**Remember:** The sum of the lengths of any two sides of a triangle is greater than the length of the third side .

### The Relation of two variables

**$a x + b y = c$  where  $a \neq 0$  ,  $b \neq 0$**  is called a linear relation of two variable  $x$

and  $y$  and can be described by a set of ordered pairs  $(x, y)$  verifying this relation.

#### Example:

Refer to the relation  $2x - y = 1$

If  $x = 1$ ,  $\therefore y = 1$   $\therefore (1, 1)$  satisfies the relation

If  $x = 0$ ,  $\therefore y = -1$   $\therefore (0, -1)$  satisfies the relation

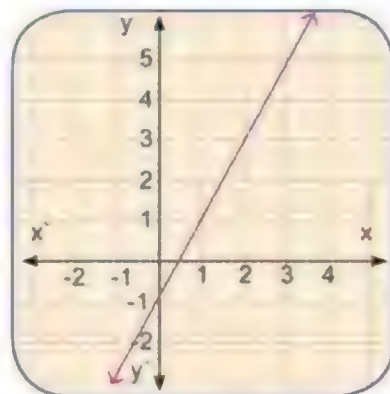
If  $x = 3$ ,  $\therefore y = 5$   $\therefore (3, 5)$  satisfies the relation

If  $x = -1$ ,  $\therefore y = -3$   $\therefore (-1, -3)$  satisfies the relation

Thus, there are an infinite number of ordered pairs satisfying the relation.

#### Note that:

- The linear relation  $2x - y = 1$ , can be represented graphically by using any of the ordered pairs obtained before.
- Each point  $\in$  the straight line (in red) is represented by an ordered pair whose elements satisfy the linear relation  $2x - y = 1$ .







### Practice

- 1 Find four ordered pairs satisfy each linear relation and represent it graphically:

A  $x + y = 3$

B  $x - 2y = 5$

C  $y = 2$

D  $x = 1$

- 2 Find the value of  $b$ , where  $(-3, 2)$  satisfies the relation  $3x + b y = 1$ .

- 3 Find the value of  $k$ , where  $(k, 2k)$  satisfies the relation  $x + y = 15$ .

### Graphing the Relation of two Variables

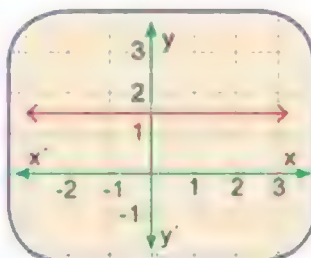
The relation  $ax + by = c$  where  $a$  and  $b$  or both are not equal zero. is called a linear relation of two variables  $x$  and  $y$  and can be represented graphically by a straight line.

for  $a = 0$

The relation is represented by a straight line parallel to  $x$ -axis.

**Example :**  $2y = 3$

i.e. :  $y = \frac{3}{2}$  is represented by the red line which passes through the point  $(0, \frac{3}{2})$  and is parallel to  $x$ -axis.



**Special case:**

the relation  $y = 0$  represents the  $x$ -axis

for  $b = 0$

The relation is represented by a straight line parallel to  $y$ -axis.

**Example :**  $x = -2$

is represented by the red line which passes through the point  $(-2, 0)$  and is parallel to  $y$ -axis.



**Special case:**

the relation  $x = 0$  represents the  $y$ -axis.



### Practice

- 1 Graph each relation of the following:

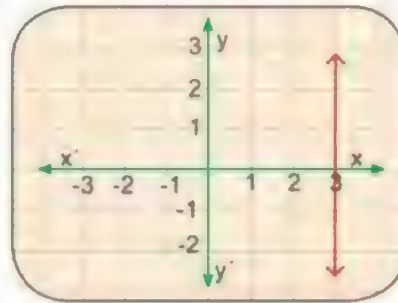
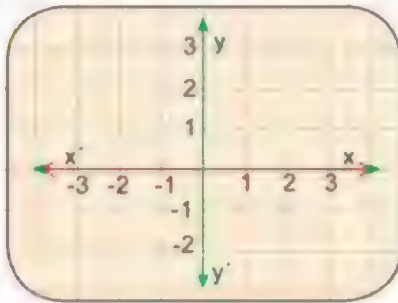
A  $2x = 5$

B  $y + 1 = 0$





- 2 Find the relation that is represented by the red line in each figure below:



**Example :**

Graph the relation:  $x + 2y = 3$

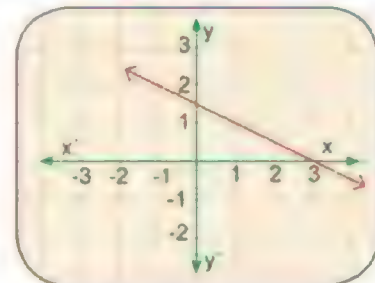
**Solution**

Choose some ordered pairs that satisfy the relation:

**Example :** For  $y = 2$   $\therefore x = -1$   $(-1, 2)$  satisfies the relation  
 $y = 0$   $\therefore x = 3$   $(3, 0)$  satisfies the relation  
 $y = -1$   $\therefore x = 5$   $(5, -1)$  satisfies the relation and so on .....

The following table lists these data:

x	-1	3	5	0
y	2	0	-1	$\frac{3}{2}$



The red line represents this relation.

**Discuss with your teacher:**

- 1 What happens to the value of  $y$  when increasing the value of  $x$ ?
- 2 When does the line representing the relation  $ax + by = c$  pass through the origin 0?

## Exercises (2 – 1)

- 1 Graph each of the following:

A  $x + y = 2$

B  $2x - y = 3$

- 2 Graph the relation  $2x + 3y = 6$ . If the straight line representing this relation intersects the  $x$ -axis in point A and the  $y$ -axis in point B. Find the area of the triangle OAB, where O is the origin.



# Unit Two

## Lesson Two

## The Slope of a line and real-life Applications

### Think and Discuss

#### You will learn how

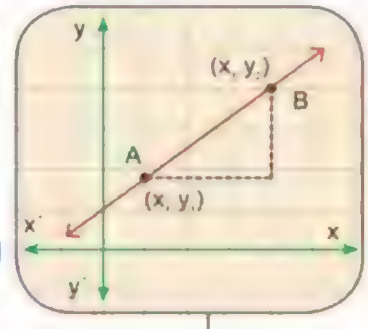
- ☞ The slope of a line .
- ☞ Real-life applications on the slope of a line.

#### key terms

- ☞ Slope.
- ☞ Positive slope.
- ☞ Negative slope.
- ☞ Zero-slope.
- ☞ Undefined slope.

When observing the motion of a point on a straight line from the location A  $(x_1, y_1)$  to the location B  $(x_2, y_2)$ , where  $x_2 > x_1$  and A, B  $\in$  line, then:

- 1 the change in x-coordinate  $= x_2 - x_1$ , and is called the horizontal change.
- 2 the change in y-coordinate  $= y_2 - y_1$  is called the vertical change and may be positive, negative or zero.



The slope of a line =  $\frac{\text{change in y-coordinate}}{\text{change in x-coordinate}} = \frac{\text{vertical change}}{\text{horizontal change}}$

$$S = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_2 \neq x_1$$

In the following examples you will learn different cases of the vertical change  $(y_2 - y_1)$  :

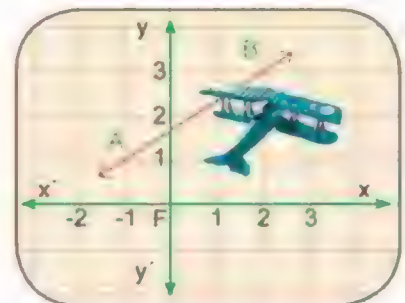


#### Example (1)

If: A  $(-1, 1)$  and B  $(2, 3)$ ,

then: the slope of  $\overleftrightarrow{AB}$

$$= \frac{3-1}{2-(-1)} = \frac{2}{3}$$





**Note that :**

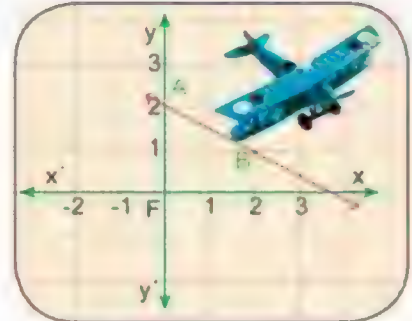
- 1 The point A moves on the line upwards to the point B.
- 2  $y_2 > y_1$
- 3 The slope of the line is positive.



**Example (2) :**

If: A (0, 2), B (2, 1);

then: the slope of  $\overleftrightarrow{AB} = \frac{1-2}{2-0} = -\frac{1}{2}$



**Not that :**

The point A moves on the line downwards to the point B

- 2  $y_2 < y_1$
- 3 The slope of the line is negative.

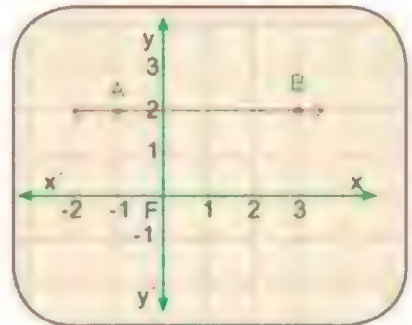


**Example (3) :**

If: A (-1, 2) and B (3, 2).

then: the slope of the line

$$\overleftrightarrow{AB} = \frac{2-2}{3-(-1)} = \frac{0}{4} = 0$$



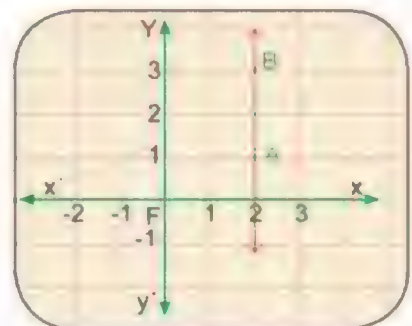
**Not that :**

- 1 The point A moves horizontally to point B.
- 2  $y_2 = y_1$
- 3 The slope of the line = zero



**Example (4) :**

If: A = (2, 1) and B(2, 3) then: we can not calculate the slope. Because the definition of the slope is conditioned to have a change in the x-coordinate i.e.  $x_2 - x_1 \neq 0$



**Not that :**

- 1 The point A moves vertically to point B.
- 2  $x_2 = x_1$
- 3 The slope of the line is an underfined number.





### Practice :

1 Find the slope of the straight line  $\overleftrightarrow{AB}$  in each of the following cases:

A  $A(1, 2), B(5, 0)$ .

B  $A(2, -1), B(4, -1)$ .

C  $A(-1, 3), B(2, 1)$ .

D  $A(3, -1), B(3, 2)$ .

2 Find the slope of  $\overleftrightarrow{AB}$ ,  $\overleftrightarrow{BC}$  and  $\overleftrightarrow{AC}$ , where  $A(2, -1)$ ,  $B(3, 2)$ , and  $C(4, 5)$  and represent each line graphically. What do you observe?

3 Choose the true answer:

**First:** The following table shows the relation between  $x$  and  $y$  as follows:

$x$	1	2	3	4	5
$y$	1	3	5	7	9

( $y = x + 4$  or  $y = x + 1$  or  $y = 2x - 1$  or  $y = 3x - 2$ )

**Second:** If  $(2, -5)$  satisfies the relation  $3x - y + c = 0$ , then  $c =$  (1, -1, 11, -11)

**Third:**  $(3, 2)$  does not satisfy the relation ( $y + x = 5$ ,  $3y - x = 3$ ,  $y + x = 7$ ,  $y - x = 1$ )

**Fourth:** An irrigation machine consumes 2.47 Litres of diesels to work for 3 hours. If the machine works for 10 hours, it consumes.... litres. (7.2, 8, 8.4, 9.6)

4 Find the slope of the line  $\overleftrightarrow{AB}$ , where  $A(-1, 3)$  and  $B(2, 5)$ .

Is the point  $c(8, 1) \in \overleftrightarrow{AB}$ ?

### Real-life Applications on the slope of a line.

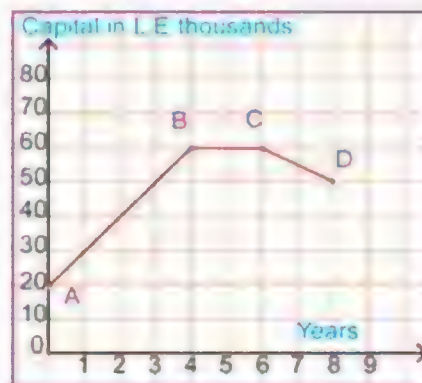
#### Application (1) :

The opposite figure shows capital change of a company during 8 years.

A Find the slope of  $\overleftrightarrow{AB}$ ,  $\overleftrightarrow{BC}$  and  $\overleftrightarrow{CD}$

What is the meaning of each?

B Find the starting capital of the company.



#### Solution

$A(0, 20)$ ,  $B(4, 60)$ ,  $C(6, 60)$ ,  $D(8, 50)$





## Unit 2 : Lesson 2

**First:** The slope of  $\overleftrightarrow{AB} = \frac{60 - 20}{4 - 0} = 10$ , shows the increasing of the capital during the first four years with a rate of 10 thousand pound.

The slope of  $\overleftrightarrow{BC} = \frac{60 - 60}{6 - 4} = 0$ , means that the capital was constant during the fifth and sixth years.

The slope of  $\overleftrightarrow{CD} = \frac{50 - 60}{8 - 6} = -5$  shows the decreasing of the capital during the last two years with a rate of 5 Thousand pound

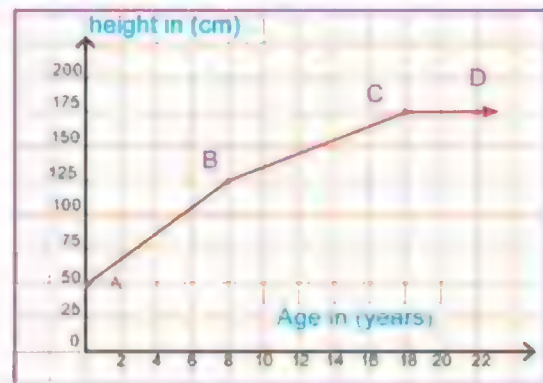
**Second:** Starting Capital = the y-coordinate of the point A = LE 20,000



### Practice:

The opposite figure shows the relation between the height of a person (in cm) and his age (in years).

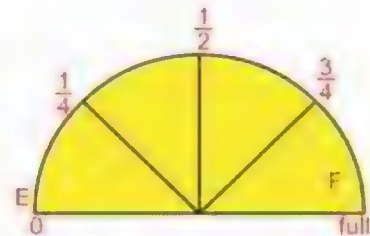
**First:** Find the slope of  $\overleftrightarrow{AB}$ ,  $\overleftrightarrow{BC}$  and  $\overleftrightarrow{CD}$   
What is the meaning of each?



**Second:** Calculate the difference between the height of this person as he was 8 years old and his height as he was 30 years old.

### Application (2) :

Hazem filled up the 40 Litres tank of his car. As covering a distance of 120 km, the fuel gage shows the rest of fuel is  $\frac{3}{4}$  of the tank. Draw a diagram to show the relation between the amount of fuel in the tank and covered distance (This relation is linear). Calculate the covered distance as the tank is totally getting empty.



### Solution

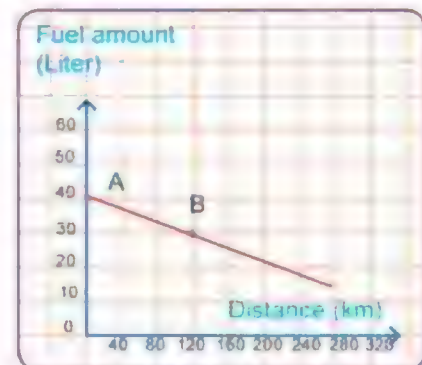
**On the starting point: A (0, 40)**

traveled distance      the amount of used fuel

After covering 120 km      B = ( 120, 30)

The slope of  $\overleftrightarrow{AB} = \frac{30 - 40}{120 - 0} = \frac{-1}{12}$

This slope means the fuel amount decreases with a rate of 1L per 12 km, which means 1L is enough to cover a distance of 12 km.



$$\begin{aligned} \text{The covered distance that make the tank empty} &= \frac{\text{Fuel Amount}}{\text{Decreasing Rate}} = \frac{40}{\frac{1}{12}} \\ &= 40 \times \frac{12}{1} = 480\text{km.} \end{aligned}$$

**Note that:**

$\overrightarrow{AB}$  intersects the distance-axis in the point (480, 0) which gives the required distance.

## Exercises (2 – 2)

1 Complete to make a true statement:

- A Let A (1, 3) and B (2, 1). Then the slope of  $\overleftrightarrow{AB}$  =
- B If (-1, 5) satisfies the relation.  $3x + ky = 7$ , then k =
- C The slope of any line parallel to x-axis =
- D The slope of any line parallel to y - axis =
- E If A, B and C are collinear points. Then the slope of  $\overleftrightarrow{AB}$  = the slope of

2 Essam has 10 bills of LE 5 and other bills of LE 20 . He bought some goods from a shopping center for LE 65. Determine the different possibilities to pay this amount of money. Find the relation and graph it.

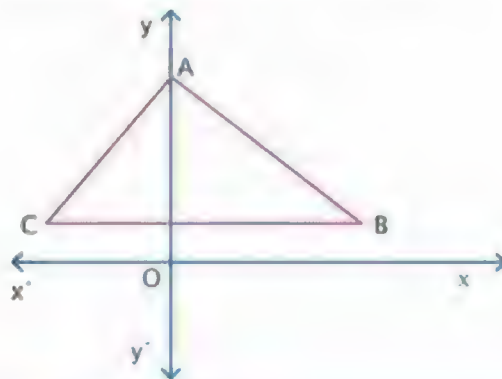
3 The selling price of a computer table is LE 100 and its chair is LE 50. If the store sells in one week with LE 500. What are the represented expectations to the number of computer tables and chairs?

Represent the relation graphically.

4 In the opposite figure ABC is a triangle. Complete by using one of the following words:

(positive, negative, zero, undefined)

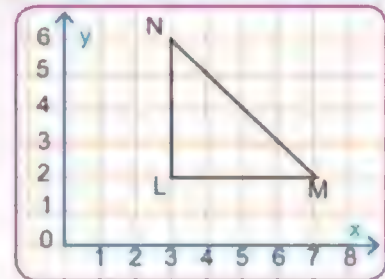
- A The slope of  $\overleftrightarrow{AB}$  .....
- B The slope of  $\overleftrightarrow{BC}$  .....
- C The slope of  $\overleftrightarrow{AO}$  .....
- D slope of  $\overleftrightarrow{AC}$  .....



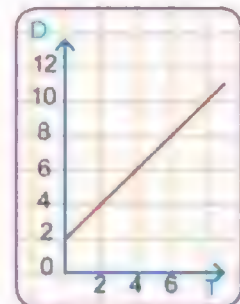
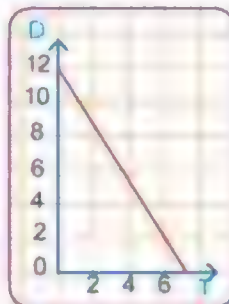
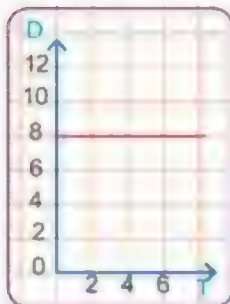


5 In the opposite figure:


LMN is a right angled triangle at L, where  $m(\angle M) = 45^\circ$ , given that L (3, 2) and M (7, 2). Find the coordinates of N and calculate the slope of  $\overleftrightarrow{MN}$ .




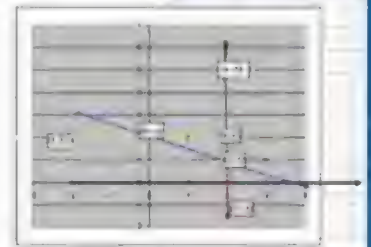
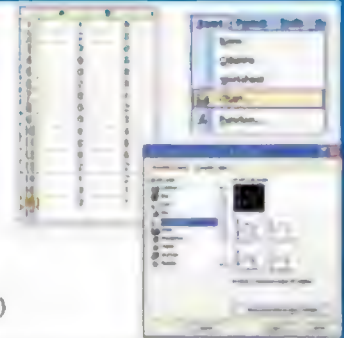
6 The following diagrams shows the relation between the coverd distance (in m) and the elapsed time (in sec) of an object. Determine the position of the object at the starting motion and its position after 6 seconds when  $t = 6$  sec. Find the slope of the line in each case, and state what it represents.



## Technology:

- 1 open micro soft office Excel to draw the two axes x-y then write the shown number as in fig(1) in the first column A and column B.
- 2 Do a mouse click to shade the two columns then from the menue INSERT choose CHART as in fig (2) then XY SCATTER as in fig (3) then press NEXT and FINISH, the x-y axes appear.
- 3 Do a mouse click on  from the menue of drawing downward of the page EXCEL and determine the values of dots as shown in fig (4)

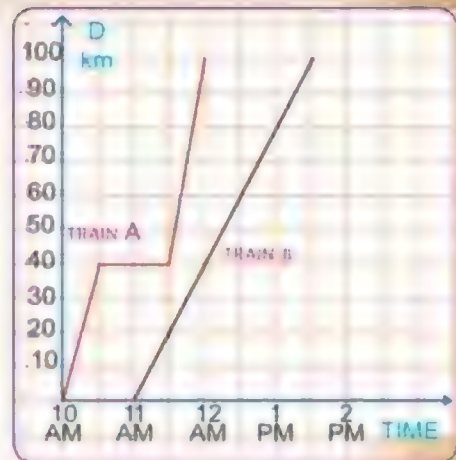
- 4 Do a mouse click on 
  - A Draw a straight line passes through (2, 1) and (0, 2) then the slope is equal to  $(2 - 1) \div (0 - 2)$  which is equal to  $-\frac{1}{2}$  of the blue line.
  - B Draw a straight line passes through (2, 2) and (-2, 2), then the slope is equal to  $(2 - 2) \div (-2 - 2)$  which is equal to zero. i.e. the line is parallel to the x axis- the yellow line.
  - C Draw a straight line passes through (2, -1) and (2, 5), then the slope is equal to  $(5 - (-1)) \div (2 - 2)$ , then the slope is undefined, i.e. the line is parallel to the y-axis - the red line.



## Activity

The opposite diagram shows the relation between the covered distance (in km) and elapsed time (in h) of two trains A and B over the distance between two train stations. Use the diagram to find:

- A The distance between the two train stations.
- B The elapsed time of each train.
- C The average speed of each train.
- D The meaning of the horizontal segment in the diagram of train A.



The average speed =  $\frac{\text{The covered distance}}{\text{Total elapsed time needed to cover this distance}}$

## Unit Test

1 Choose the true answer:

- A Which of the following ordered pairs satisfies the relation  $2x + y = 5$   
 $((-1, 3), (1, 3), (3, 1), (2, 2))$

- B Which of the following relations is the relation illustrated in the table.  
 $(y = x + 7, y = x - 7, y = 3x + 1, y = x + 1)$

X	3	4	5
Y	10	13	16

- C Let A (3, 5) and B (5, -1), then the slope of  $\overleftrightarrow{AB} = (-\frac{1}{3}, -3, 3, \frac{1}{3})$

- D The line represented by the relation  $3x + 8y = 24$ , intersects the y-axis at the point  $((0, 8), (8, 0), (0, 3), (3, 0))$

2 Let A (2, -1), B (10, 3) and C (2, 3). Find the slope of  $\overleftrightarrow{AB}$ ,  $\overleftrightarrow{BC}$  and  $\overleftrightarrow{AC}$

**Draw the triangle ABC** on a square grid, then tell the type of the triangle according to its angles.

3 Atef filled up the 50 L-tank of his car. As covered a distance of 100km, the fuel gage shows the rest of fuel is  $\frac{4}{5}$  of the tank. Draw a diagram to show the relation between the amount of fuel remaining in the tank and the distance covered. What is the covered distance as the tank is getting totally empty?





## UNIT THREE

### 3

# Statistics



## UNIT THREE

### Lesson One

## Collecting and Organizing data

### Think and Discuss

#### You will learn how

- To collect and organize data
- Using frequency tables with sets

#### key terms

- Collecting data
- Organizing data.
- Frequency table with sets

If you study the traffic jam problem and its possible solutions:

- 🚗 What are the sources of your data?
- 🚗 How can you collect data about such a problem?
- 🚗 What are the statistical methods you will use to analyze the data?
- 🚗 Can you explain the results you collected?
- 🚗 What do you suggest to solve that problem and improve traffic fluidity?



### Collecting data

**Let's work together** Cooperate with your classmates on collecting data from their sources through distribution of roles:

- A Group 1:** Collects primary data about the problem under discussion through a survey that asks about (the means of transportation - Roads conditions - time of traffic jam - Existence of traffic signs - existence of security).
- B Group 2:** Collects secondary data about the problem under discussion from the traffic reports - the internet - the mass media).
- C Group 3:** Observes the crowdest roads, the drivers' behavior and their obedience to traffic rules the pedestrians' commitment to the virtues to the road as well as crossing the roads at safe places.





## Organizing and Analyzing data

Cooperate with your classmates on making a frequency table that represents the means of transportation used by your classmates..

Means of transportation	Subway	bus	Private Car	Taxi	bicycle	on foot	total
Frequency	.....	.....	.....	.....	.....	.....	.....

**Determine the most used means of transportation (The mode)**

1. Is that means suitable? does it help solving the traffic jam problem? why?
2. What do you suggest to solve this problem according to the results you have collected?

## Organizing data and representing them in frequency tables



### Example

Below are the scores of 30 students in an examination

7	10	7	4	5	8	6	7	13	12
2	9	11	12	11	9	15	12	13	9
5	14	19	3	9	14	3	13	8	17

**Required:** forming a frequency table with sets that represents that data .

### Solution

To form a frequency table with sets, follow the following steps:

**First:** find the highest and the lowest values of the collected data?

let the previous collected data be X

**then:**  $X = \{x : 2 \leq x \leq 19\}$

**i.e:** X values begins with 2 and ends in 19

**i.e:** the range = the highest value - the lowest value =  $19 - 2 = 17$

**Second:** divide set X into a number of separate subsets each of them is equal in range.

let them be 6 sets.  $\therefore$  The range of the set =  $\frac{17}{6}$  i.e approximated to 3



**Third:** the subsets are as follow.

The first set 2 -

The second set 5 -

the third set 8 -

The Fourth set 11 - and so on

**Remark :** 2- means the set of data greater than or equal to 2 and less than 5 and so on.

**Fourth:** Record the data in the following table:

Set	tally	frequency
2 -	////	4
5 -	//// /	6
8 -	//// //	7
11 -	//// ///	8
14 -	///	3
17 -	//	2
Total		30

**Fifth:** Delete the tally column from the table to get the frequency table with sets. It can be written either vertically or horizontally. The following is the horizontal form of the table:

Sets	2 -	5 -	8 -	11 -	14 -	17 -	total
Frequency	4	6	7	8	3	2	30

## Exercises (3 - 1)

**1** Below are the weekly wages of 40 workers in a factory in L.E

47	71	36	94	54	64	87	89	62	57
51	61	44	52	70	66	56	32	69	36
79	48	77	90	65	99	96	67	60	55
95	75	81	84	78	38	49	94	48	59

Required : Form a frequency table with sets (use the subsets: 30-, 40-, 50-, .....90-).  
What is the set with the highest frequency? what is the set with lowest frequency?





- 2 The following are the scores of 30 students in a monthly math exam.

25	35	40	20	30	37	40	33	22	38
35	36	28	37	39	28	32	26	29	37
23	34	35	36	29	38	40	35	37	31

Required:

- A Form a frequency table with sets for these scores.  
 B Find the total number of excellent students. The excellence rate is 36 marks or more.

- 3 The following table shows the days-off which 40 workers got during a year.

15	30	26	14	28	13	25	14	27	11
24	16	21	16	15	22	21	17	21	29
26	21	15	20	30	24	20	20	15	26
29	30	20	27	22	26	22	28	30	15

Required:

- A Form a frequency table with sets to represent the data above.  
 B Find the number of workers who got more than 20 days-off a year long.



# UNIT THREE

## Lesson Two

### The Ascending and Descending Cumulative Frequency Table and Their Graphical Representation

#### Think and Discuss

#### You will learn how

- ✎ To Form both ascending and descending cumulative frequency tables.
- ✎ To represent both ascending and descending cumulative frequency tables graphically.

#### key terms

- ✎ The frequency distribution.
- ✎ The frequency table.
- ✎ The ascending cumulative frequency table.
- ✎ The descending cumulative frequency table.
- ✎ The ascending cumulative frequency curve.
- ✎ The descending cumulative frequency curve.

First: Ascending cumulative frequency table and its graphical representation.



#### Examples

The following table shows the frequency distribution for the heights of 100 students in a school in centimeters.

Tall (sets) in c.m	115–	120–	125–	130–	135–	140–	145–	Total
Number of students (frequency)	8	12	19	23	18	13	7	100

- 1 How many students are with height less than 115cm?
- 2 How many students are with height less than 135cm?
- 3 How many students are with height less than 145cm?

Form the ascending cumulative frequency table for these data and represent them graphically.

#### Solution

- Are there students with height less than 115c.m? **No**
- Are there students with height less than 135c.m? How many? **yes, 62 student.**
- How can you calculate the number of students with height less than 145 cm? **Add the number of students in the sets of height less than the set 145.**

Now, to answer the previous questions in an easier way, form an ascending cumulative frequency table as follows:





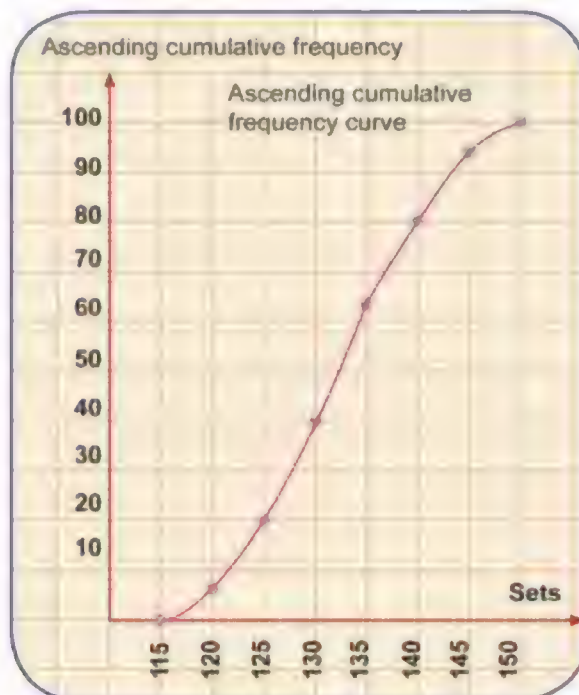
Upper boundaries of sets	Ascending cumulative frequency
Less than 115	0
Less than 120	0 + 8 = 8
Less than 125	8 + 12 = 20
Less than 130	20 + 19 = 39
Less than 135	39 + 23 = 62
Less than 140	62 + 18 = 80
Less than 145	80 + 13 = 93
Less than 150	93 + 7 = 100

i.e.

ascending cumulative frequency table	
Upper boundaries of sets	Ascending cumulative frequency
Less than 115	zero
Less than 120	8
Less than 125	20
Less than 130	39
Less than 135	62
Less than 140	80
Less than 145	93
Less than 150	100

To represent the ascending cumulative frequency table graphically:

- 1 Specify the horizontal axis to the sets and the vertical axis to the ascending cumulative frequency
- 2 Choose a drawing scale to draw the vertical axis such that the ascending cumulative frequency axis can hold the number of elements in a set
- 3 Represent the ascending cumulative frequency for each set and draw its line graph successively.



## Second: The descending cumulative frequency table and its graphical representation. :

Of the previous frequency distribution which shows the heights of 100 students in a school in centimeters.

**Find:** The number of students with heights of 150cm and more..

The number of students with heights of 140cm and more..

The number of students with heights of 125cm and more..

Form the descending cumulative frequency table and represent it graphically..

### Solution

There are no students with heights of 150cm and more .

The number of students with heights of 140cm and more is  $7 + 13 = 20$  students.

The number of students with heights of 125cm and more is

**complete:**  $19 + \dots + \dots + \dots + \dots = \dots$

To answer these questions in an easier way, form the descending cumulative frequency table as follows :

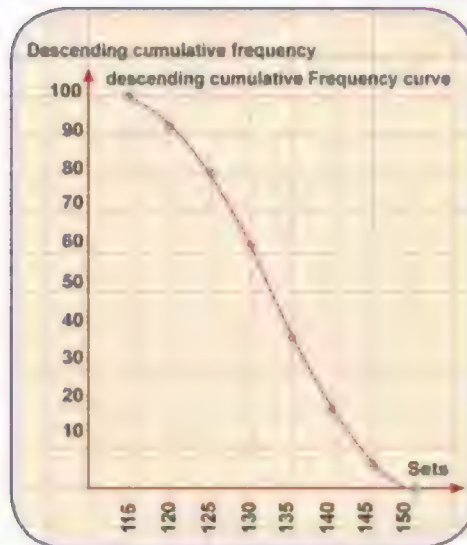
Descending cumulative frequency table	
Lower limits of sets	Ascending cumulative frequency
115 and more	100
120 and more	92
125 and more	80
130 and more	61
135 and more	38
140 and more	20
145 and more	7
150 and more	zero

Lower limits of sets	descending cumulative frequency
115 and more	$92 + 8 = 100$
120 and more	$80 + 12 = 92$
125 and more	$61 + 19 = 80$
130 and more	$38 + 23 = 61$
135 and more	$20 + 18 = 38$
140 and more	$7 + 13 = 20$
145 and more	$0 + 7 = 7$
150 and more	0

To represent this table graphically, follow the steps of representing the ascending cumulative frequency to get the following graphical representation:







## Exercises (3 – 2)

- 1 Below are the scores of 100 students in an experimental Maths exam.

Sets	0–	10–	20–	30–	40–	50–	Total
Frequency	8	14	15	28	23	12	100

Required:

- A Form both the ascending and descending cumulative frequency tables.
  - B Graph both the ascending and the descending cumulative frequency curves on the same graph paper.
  - C From the graph, find the number of students who got less than 40 marks and those who got 40 marks and more.
  - D Find the percentage of success, given that the minimum mark of success is 20 marks.
  - E What is the percentage of the students who got more than 40 marks?
- 2 The following table shows the frequency distribution of the scores of 50 students in an experimental math exam.

Intervalle	2–	6–	10–	14–	18–	22–	26–	Total
Effectif	3	5	9	10	12	7	4	50

Required: Graph the ascending cumulative frequency curve.



- 3 The following table shows the frequency distribution of the daily wages of some workers.

Sets	5-	10-	15-	20-	25-	30-	Total
Frequency	10	14	24	30	12	10	100

**Required:** Graph the descending cumulative frequency curves.

- 4 The following table shows the frequency distribution of the ages of 50 workers.

Sets	20-	25-	30-	35-	40-	45-	50-	Total
Frequency	5	8	9	13	....	5	3	50

**Required:**

- A Complete the missing space.
- B Graph the ascending and descending cumulative frequency curves.
- C Find: First: the number of workers whose ages are more than 32.

Second: The number of workers whose ages are less than 43.

- 5 The following table shows the frequency distribution of the scores of 1000 students in a final year exam.

Percentages	20-	30-	40-	50-	60-	70-	80-	90-	Total
Number of students	30	70	160	260	150	130	110	90	1000

**Required:**

- A Graph the ascending and descending cumulative frequency curves.
- B Find the number of students whose scores are less than 75 marks.
- C Find the number of students whose scores are more than 85 marks.





Arithmetic Mean, Median and  
Mode

## Think and Discuss

## First: the mean

You have learned to find the mean for a set of values and learned that:

$$\text{The arithmetic mean} = \frac{\text{The sum of values}}{\text{Number of values}}$$

**Example:** If the ages of 5 students are 13, 15, 16, 14, and 17 years old, then

$$\begin{aligned}\text{The mean of their ages} &= \frac{13 + 15 + 16 + 14 + 17}{5} \\ &= \frac{75}{5} = 15 \text{ years}\end{aligned}$$

**Remark:**  $15 \times 5 = 13 + 15 + 16 + 14 + 17$

**The mean:** is the simplest and most commonly used type of averages. It's that value given to each item in a set, then the total of these new values is the same total of the original values. It can be calculated by adding up all values, then divide the sum by the number of values.

**Finding the mean of data from the frequency table with sets:**

How can you find the mean of the following frequency distribution:

Sets	10 -	20 -	30 -	40 -	50 -	Total
Frequency	10	20	25	30	15	100

**Remark:** To find the mean for a frequency distribution with sets, follow the following steps:

## You will learn how

- ✎ To find the mean from a frequency table with sets.
- ✎ To calculate the median from a frequency table.
- ✎ To calculate the mode from a frequency table with sets.

## key terms

- ✎ Mean.
- ✎ Median.
- ✎ Frequency histogram.
- ✎ Mode.



**1 Determine the centers of sets:**

The center of the first set =  $\frac{20 + 10}{2} = 15$ . The center of the second set =  $\frac{30 + 20}{2} = 25$  ... and so on

Since the ranges of the subsets are equal and each = 10

We consider the upper limit of the last set = 60 and then :

$$\text{its center} = \frac{50 + 60}{2} = 55$$

**2 Form the following vertical table:**

Sets	Centre of the sets (X)	Frequency	Centre of the sets X	× frequency F
10 -	15	10		150
20 -	25	20		500
30 -	35	25		875
40 -	45	30		1350
50 -	55	15		825
Total		100		3700

**3 The mean =**  $\frac{\text{The total of } (F \times X)}{\text{the total of } F}$

$$= \frac{3700}{100} = 37$$



**Practice**

- 1 If the mean of the scores of a student during the first 5 months is 23.8. What is the score of the 6<sup>th</sup> month If the mean of his scores is 24 marks?
- 2 The following table shows the frequency distribution of the weights of 30 children in kg.

Weight in (kg)	6-	10-	14-	18-	22-	26-	30-	Total
frequency	2	3	....	8	6	4	2	30

Complete the table, then find the mean of such a distribution.





### Second: the median

The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it.

#### Finding the median of a frequency distribution with sets graphically:

- 1 Draw the ascending or descending cumulative frequency table, then draw the cumulative frequency curve of it .
- 2 Determine the order of the median =  $\frac{\text{The total of frequency}}{2}$  .
- 3 Determine point A on the vertical axis (frequency) which represents the order of the median.
- 4 Draw a horizontal straight line from point A to intersect the curve at a point. From this point, draw a vertical straight line on the horizontal axis to intersect it at a point that represents the median.



#### Example (1)

The following table shows the frequency distribution for the scores of 60 students in an exam.

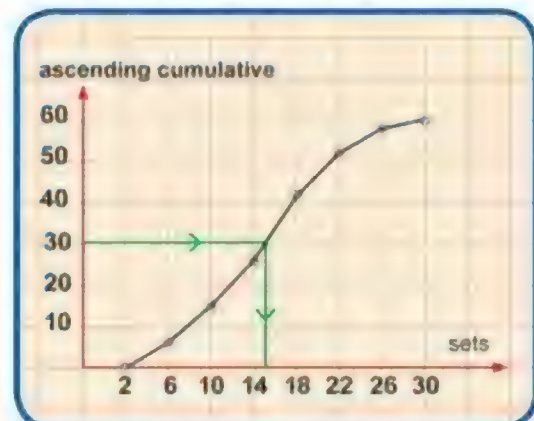
Sets	2-	6-	10-	14-	18-	22-	26-	Total
Frequency	6	9	12	15	10	5	3	60

Find the median of the distribution using the ascending cumulative frequency table.

#### Solution

- 1 Draw an ascending cumulative frequency table.
- 2 Find the order of the median =  $\frac{60}{2} = 30$
- 3 Draw the ascending cumulative frequency curve, and get the median from the graph.

The upper limits of the sets	The ascending cumulative frequency
Less than 2	0
Less than 6	6
Less than 10	15
Less than 14	27
Less than 18	42
Less than 22	52
Less than 26	57
Less than 30	60



From the graph, the median = 14.8 mark





**Think up** Can you find the median using the descending cumulative frequency table? Is the value of the median different in such a case?.



### Example (2)

The following table shows the daily wages of 100 workers in a factory..

daily wages in LE (sets)	15–	20–	25–	30–	35–	40–	Total
Number of workers (frequency)	10	15	22	25	20	8	100

**Required:**

- 1 Graph the ascending and descending cumulative frequency curves on one figure.
- 2 Can you find the median wage from this curve?

**Solution**

Upper boundaries of sets	Cumulative frequency	Lower boundaries of sets	Cumulative frequency
Less than 15	zero	15 and more	100
Less than 20	10	15 and more	90
Less than 25	25	15 and more	75
Less than 30	47	15 and more	53
Less than 35	72	15 and more	28
Less than 40	92	15 and more	8
Less than 45	100	15 and more	zero

**Remark:**

The ascending cumulative frequency curve intersects with the descending cumulative frequency curve at one point which is m .





## Unit 3 : Lesson 3

The y-coordinate for the point M = 50  
 $= \frac{100}{2}$

= the order of the median

∴ **The X-coordinate of the point M determines the median**

every 10 mm of the x coordinate represents L.E 5

**Complete: 2 mm represents .....**

The median wage =  $30 + \frac{2 \times 5}{10} = \text{LE } 31$ .



**Draw the descending cumulative**

**frequency curve for the following frequency distribution, then find the value of the median.**

Sets	5 -	10 -	15 -	20 -	25 -	30 -	total
Frequency	4	6	10	17	10	3	50

**Third: the mode**

The mode is the most common value in the set or in other words, it is the value which is repeated more than any other values.



**Example**

The following table shows the frequency distribution for the scores of 40 students in an examination.

Sets	2-	6-	10-	14-	18-	22-	26-
Frequency	3	5	8	10	7	5	2

Find the mode of this distribution graphically

**Solution**

You can find the mode of this distribution graphically using the histogram as follows:

**First: draw a histogram.**

- 1 Draw two perpendicular axes: one horizontal to represent sets and the other vertical to represent the frequency of each set.

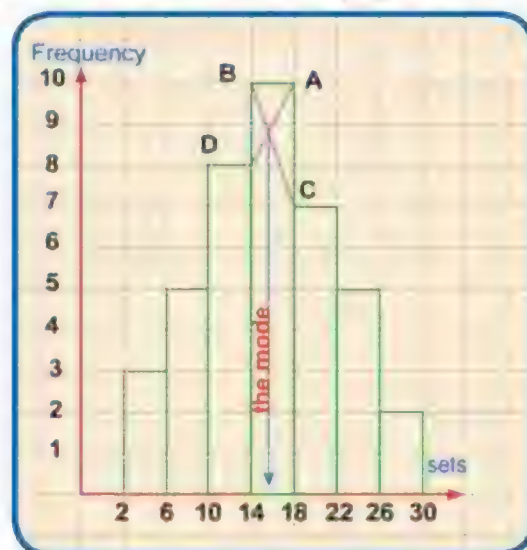


- 2 Divide the horizontal axis into a number of equal parts using a suitable drawing scale to represent sets.
- 3 Divide the vertical axis into a number of equal parts using a suitable drawing scale such that the greatest frequency among sets can be represented..
- 4 Draw a rectangle whose base is set (2-) and height is equal to the frequency (3).
- 5 Draw another rectangle adjacent to the first one whose base is set (6-) and height is equal to the frequency (5).
- 6 Repeat drawing the rest of adjacent rectangles till the last set (26-).

**Second: Finding the mode from the histogram,** to find the mode from the histogram, we observe that: the most repeated set is (14-), and it is called the mode set, why?

Define the intersection point of  $\overline{AD}$  ,  $\overline{BC}$  from the graph, and from this point, drop a vertical line on the horizontal axis to define the sequential value within that distribution.

From the graph, what's the mode value?



### Exercises (3 – 3)

- 1 The following table shows the frequency distribution of 50 workers days-off:

Sets	2-	6-	10-	14-	18-	22-	26-
Frequency	4	5	8	K-2	7	5	1



**Find**

- A the value of K
- B The Arithmetic mean for that distribution.

- 2 The following table shows the frequency distribution of the heights of 120 students in centimeters:

Height in (cm)	140-	144-	148-	152-	156-	160-	total
Frequency	12	20	38	22	17	11	120

Find the mean.





- 3 The following table shows the frequency distribution of 50 workers' wages in a factory:

sets of wages	300-	400-	500-	600-	700-	total
Number of workers	8	12	18	7	5	50

Graph the descending cumulative frequency curve, then find the median.

- 4 From the following frequency table with equal sets in range.

Sets	10-	20-	30-	40-	x -	60-	total
Frequency	12	15	25	27	k + 4	4	100

- A The value of both X and K.  
B Graph the ascending and descending cumulative Frequency curves in one figure, then find the median.

- 5 The following table shows the frequency distribution of the weights of 50 students in Kg.

weight in kg	30-	35-	40-	45-	50-	55-	total
Number of students	k + 4	3k	4k	3k + 1	3k - 1	k + 1	50

 Find

- A the value of K.  
B Graph the frequency histogram, then find the mode weight.

- 6 The following table shows the frequency distribution of the height of 200 students.

Height in (cm)	110-	115-	120-	125-	130-	135-	140-	total
Number of students	10	12	28	35	60	40	40	200

Graph the frequency histogram, then find the mode length.



## General Exercises

- 1 The following table shows the frequency distribution for the scores of 50 students in an examination:

Sets	2 -	6 -	10 -	14 -	18 -	22 -	26 -	total
Frequency	3	5	9	10	12	7	4	50



**Find** First: the mean of the student's score. Second: The median

- 2 From the following frequency table with equal sets in range, find:

Sets	10 -	20 -	x -	40 -	50 -	60 -	total
Frequency	10	17	20	32	K + 2	4	100

First: find the value of X and K:

Second: graph the ascending and descending cumulative curves on one figure, then calculate the median

- 3 **Find** the mode of the following frequency distribution for the scores of 40 student in an examination:

Sets of marks	30 -	40 -	50 -	60 -	70 -	80 -	total
frequency	3	4	12	8	7	6	40

- 4 The following table shows the frequency distribution with equal - range sets for the weekly wages of 100 works in a factory.

Sets of wages in L.E	70 -	80 -	90 -	100 -	x -	120 -	130 -
Number of workers	10	13	f - 4	20	16	14	11



- Trouver**
- A The value of x and F
  - B The mode of wages in L.E





## Activity

The following table shows the frequency distribution for the weights of 50 students in K.g at a school.

Weight in K.g	30 –	35 –	40 –	45 –	50 –	55 –	total
Number. of students	7	3 k	4k	10	8	4	50

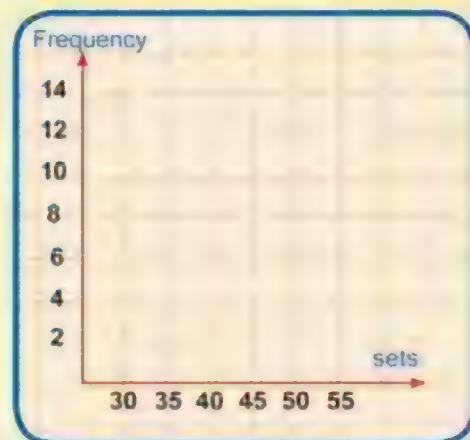
**First:** find the value of K.

**Second:** calculate the mean.

**Third:** Draw the ascending cumulative frequency curve.

**Fourth:** Draw the histogram and find the mode of weights.

**Fifth:** Find the median.



## Unit test

1 Complete the following:

- A If the lower limit of a set is 8 and the upper limit of the same set is 14, then its centre is .....
- B If the lower limit of a set is 4 and its center is 9, then its upper limit is .....
- C The intersection point of the ascending and descending cumulative frequency curves determines ..... on the sets axis
- D The location of the top of the frequency curve on the set axis is .....
- E If the mean of a frequency distribution is 39.4 and the total of its frequency is 100, then the total of the product of multiplying each set frequency by its centre = .....

2 The following table shows the frequency distribution of weights of 20 children in k.g

sets	5 -	15 -	25 -	35 -	45 -	total
Frequency	3	4	7	4	2	20

Find the median weight in k.g. using the ascending and descending cumulative frequency curve of this distribution.

3 below is the frequency distribution of the weekly bonus of 100 workers in a factory.

Bonus in L.E.	20 -	30 -	40 -	50 -	60 -	70 -
No. of workers	10	K	22	26	20	8

- A Calculate the value of K.
- B Find the mean of this distribution.
- C The mode value of the weekly bonus using the histogram..





## UNIT FOUR

### 4

## Geometry



# UNIT FOUR

## Lesson One

## Medians Of Triangle

### Think and Discuss

#### You will learn how

- Medians of the triangle.
- A  $30^\circ - 60^\circ - 90^\circ$  triangle.

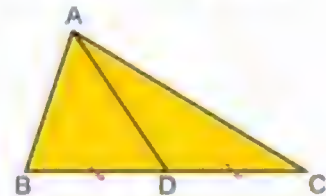
#### key terms

- Median of the triangle.
- A  $30^\circ - 60^\circ - 90^\circ$  triangle.
- Point of Concurrence

The medians of a triangle is the line segment drawn from the triangle vertex to the middle of the opposite side of this vertex.

ABC is a triangle where the point D bisects  $\overline{BC}$ .

So  $\overline{AD}$  is a triangle Median.



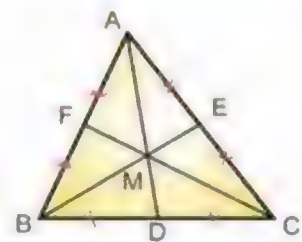
- How many medians does the triangle have?
- Draw the medians in each triangle.



#### Theorem 1

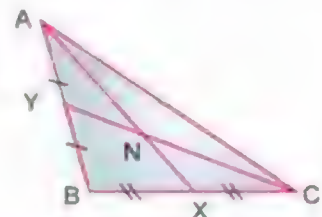
The medians of a triangle are concurrent

ABC is a triangle where point D bisects  $\overline{BC}$ , point E bisects  $\overline{AC}$ , point F bisects  $\overline{AB}$ , then  $\overline{AD}$ ,  $\overline{BE}$  and  $\overline{CF}$  all intersect in one point (M)



#### Practice

In the Figure opposite: ABC is a triangle where point x bisects  $\overline{BC}$ , point y bisects  $\overline{AB}$ , and  $\overline{AX} \cap \overline{CY} = \{N\}$ .





- 1 Draw  $\overrightarrow{BN}$  to intersect  $\overline{AC}$  at point Z, then find the lengths of  $\overline{AZ}$  and  $\overline{CZ}$  is  $AZ = CZ$ ? Reason your answer.

- 2 Measure, then complete.

$$\frac{NX}{NA} = \frac{\dots}{\dots} = \frac{\dots}{\dots}, \quad \frac{NY}{NC} = \frac{\dots}{\dots} = \frac{\dots}{\dots} = \frac{NZ}{NB} = \frac{\dots}{\dots} = \frac{\dots}{\dots}$$

- If your measurements are accurate, then  $\frac{NX}{NA} = \frac{1}{2}$ ,  $\frac{NY}{NC} = \frac{1}{2}$  and  $\frac{NZ}{NB} = \frac{1}{2}$



### Theorem 2

The point of concurrence of the medians of the triangle divides each median in the ratio of 1:2 from its base

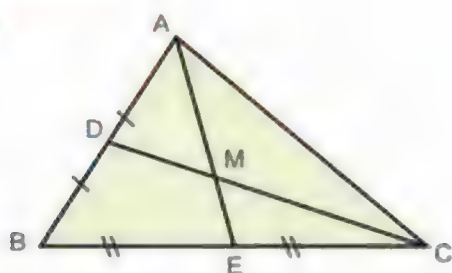


### Practice



### Complete

A

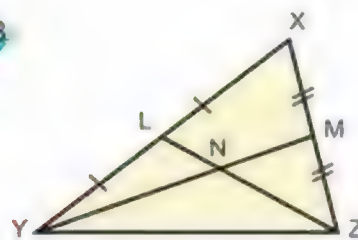


$$ME = 3\text{cm}, \quad MC = 8\text{cm}$$

$$MA = \dots, \quad MD = \dots$$

$$ME = \dots AE, \quad MC = \dots CD$$

B



$$LZ = 15\text{cm}, \quad YM = 18\text{cm}, \quad XY = 20\text{cm}$$

$$NL = \dots, \quad NY = \dots$$

$$\text{Perimeter of } \triangle NLY = \dots$$

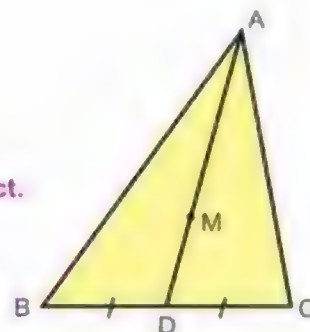
### Fact

$\overline{AD}$  is a median in  $\triangle ABC$ ,  $M \in \overline{AD}$

if  $AM = 2 MD$ ,

then

then **M is the point where the medians of the triangle intersect.**



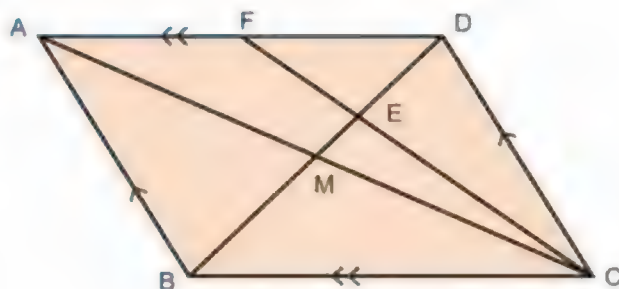


### Example (1)

In the figure opposite: ABCD is a parallelogram where its two diagonals intersect at point M, point  $E \in \overline{DM}$  and  $DE = 2 EM$ .

$\overrightarrow{CE}$  is drawn and intersected  $\overline{AD}$  at point F.

Prove that:  $AF = FD$



**Proof:** In the parallelogram ABCD

$$\therefore \overline{AC} \cap \overline{BD} = \{M\}$$

$\therefore M$  bisects  $\overline{AC}$

the triangle DAC

$\therefore M$  bisects  $\overline{AC}$

$\therefore \overline{DM}$  is a median of the triangle.

$\therefore E \in \overline{DM}$ ,  $DE = 2 EM$ .

$\therefore E$  is the intersecting point of the triangle's medians.

$\therefore E \in \overline{CF}$

$\therefore \overline{CF}$  is a median of the triangle and point F bisects  $\overline{AD}$



### Theorem 3

In the right - angled triangle, the length of the median from the vertex of the right angle equals half the length of the hypotenuse

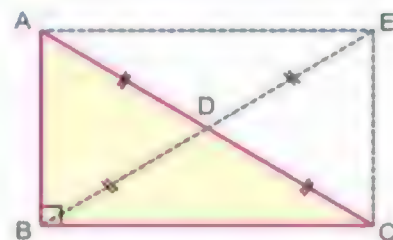
**Given data:** ABC is a triangle where  $m(\angle B) = 90^\circ$ ,  
 $\overline{BD}$  is a median in  $\triangle ABC$ .

**Required :** Prove that:  $BD = \frac{1}{2} AC$ .

**Construction :** Draw  $\overrightarrow{BD}$ , let point  $E \in \overrightarrow{BD}$  where  $BD = DE$ .

**Proof :**

$\therefore$  in the Figure ABCE,  $\overline{AC}$ ,  $\overline{BE}$  bisect each other.





∴ the Figure ABCE is a parallelogram.

∴  $m(\angle B) = 90^\circ$

∴ ABCE is a rectangle.

∴  $BE = AC$ .

∴  $BD = \frac{1}{2} BE$

∴  $BD = \frac{1}{2} AC$

Q.E.D.



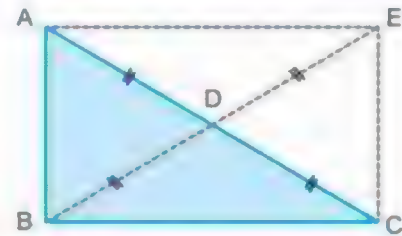
### The converse of the theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right

**Given:** ABC is a triangle where  $\overline{BD}$  is a median,  
 $BD = DA = DC$

**Required:** Prove that:  $m(\angle ABC) = 90^\circ$ .

**Construction :** Draw  $\overrightarrow{BD}$  and let point E  $\in \overrightarrow{BD}$   
 where  $BD = DE$ .



**Proof:** ∴  $BD = \frac{1}{2} BE = \frac{1}{2} AC$ .

∴  $BE = AC$ .

In the Figure ABCE, AC and BE are equal in length and bisect each other.

∴ the Figure ABCE is a rectangle.

∴  $m(\angle ABC) = 90^\circ$

Q.E.D.



### Corollary

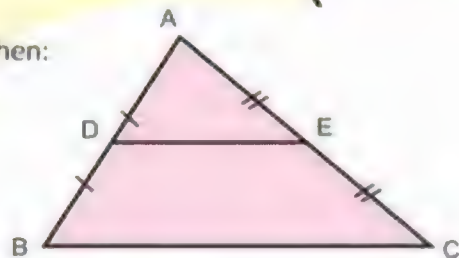
The length of the side opposite to the angle of measure  $30^\circ$  in the right - angled triangle equals half the length of the hypotenuse.

### Remember:

If the point D bisects  $\overline{AB}$  and the point E bisects  $\overline{AC}$ , Then:

①  $DE = \frac{1}{2} BC$

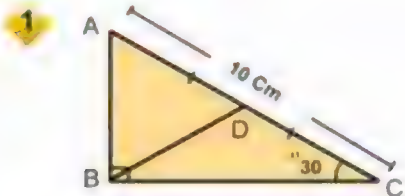
②  $\overline{DE} \parallel \overline{BC}$



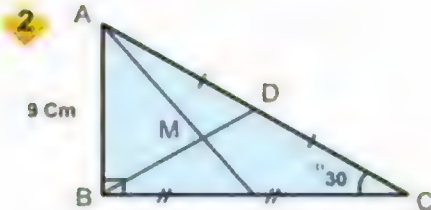
## Exercises (4-1)



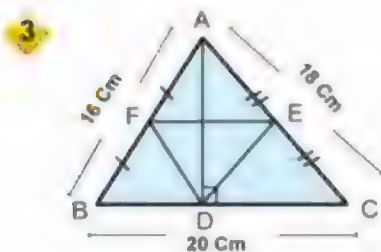
**Complete:**



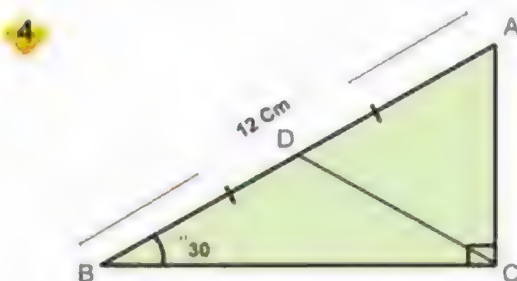
$BD = \dots\dots \text{ cm}$ ,  $AB = \dots\dots \text{ cm}$   
 Perimeter  $\triangle ABD = \dots\dots \text{ cm}$



$AC = \dots\dots \text{ cm}$ ,  $BD = \dots\dots \text{ cm}$   
 $MD = \dots\dots$   $BD, MD = \dots\dots \text{ cm}$



$DF = \dots\dots \text{ cm}$ ,  $DE = \dots\dots \text{ cm}$ ,  
 $FE = \dots\dots \text{ cm}$   
 Perimeter  $\triangle DEF = \dots\dots \text{ cm}$



$AC = \dots\dots \text{ cm}$ ,  $AD = \dots\dots \text{ cm}$   
 $BC = \dots\dots \text{ cm}$ ,  $CD = \dots\dots \text{ cm}$

5

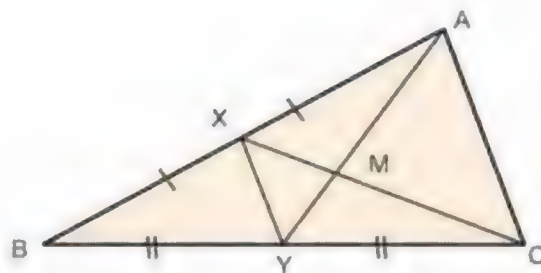
**In the figure opposite:**

$ABC$  is a triangle,  $X$  bisect  $\overline{AB}$ ,  
 $Y$  bisect  $\overline{BC}$ ,  $XY = 5 \text{ cm}$ ,  
 $\overline{XC} \cap \overline{AY} = \{M\}$   
 where:  $CM = 8 \text{ cm}$ ,  $YM = 3 \text{ cm}$



**Find:**

- (1) The perimeter of  $\triangle MX Y$
- (2) The perimeter of  $\triangle MAC$





- 6 ABC is a triangle where point D bisects  $\overline{BC}$ , and point  $M \in \overline{AD}$ .  $AM = 2 MD$ .

Draw  $\overrightarrow{CM}$  to intersect  $\overline{AB}$  at point E.

If  $EC = 12$  cm,

then find the length of  $\overline{EM}$

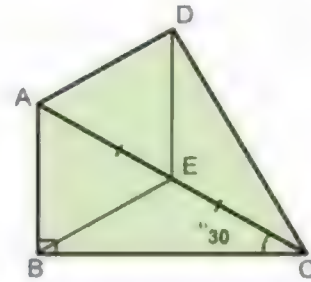
- 7 In figure opposite:

ABC is a right-angled triangle at B,

$m(\angle ACB) = 30^\circ$ .

$AB = 5$  cm, point E is the mid point of  $\overline{AC}$ .

If  $DE = 5$  cm, then prove that  $m(\angle ADC) = 90^\circ$ .



## UNIT FOUR

### Lesson Two

## The Isosceles Triangle

### Think and Discuss


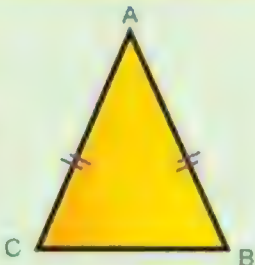
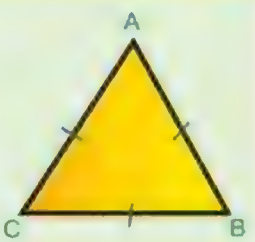
#### You will learn how

- To define the properties of the isosceles triangle.
- To define the classifications of the isosceles triangle..

#### key terms

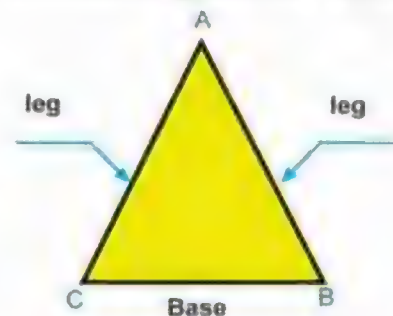
- The isosceles triangle.
- The equilateral triangle.
- The scalene triangle.

You have learnt that triangles are classified according to the lengths of their sides into three types:

The scalene triangle	The isosceles triangle (two sides are congruent)	The equilateral triangle (three sides are congruent)
 <p> <math>AB \neq BC</math>  <math>AB \neq AC</math>  <math>BC \neq AC</math> </p>	 <p><math>AB = AC</math></p>	 <p><math>AB = AC = BC</math></p>

In the figure opposite :

**Remark** the two sides  $\overline{AB}$ ,  $\overline{AC}$  are congruent (of equal lengths), so the triangle ABC is called isosceles triangle while the point A is called the vertex.  $\overline{BC}$  is the base, and the two angles B and C are the base angles of the triangle.



#### The properties of isosceles triangle

In any isosceles triangle

- What is the type of the base angles? (acute - right - obtuse)
- What is the type of the vertex angle?

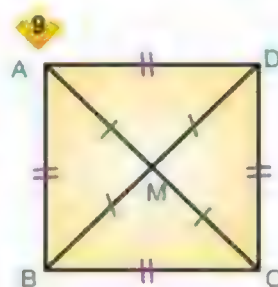
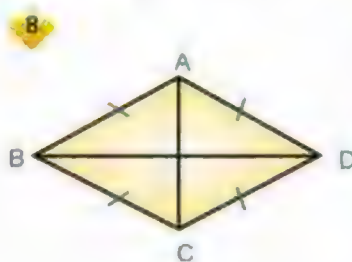
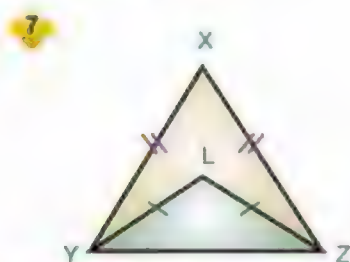
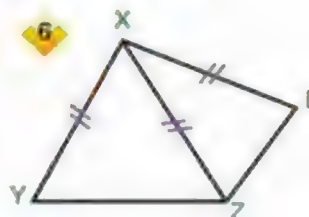
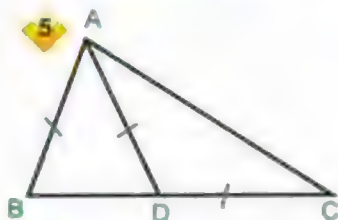
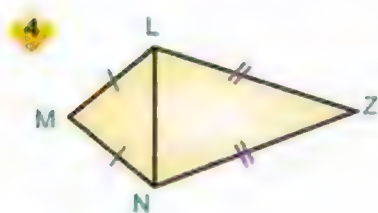
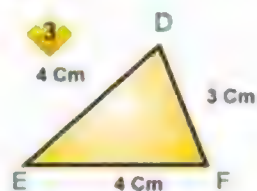
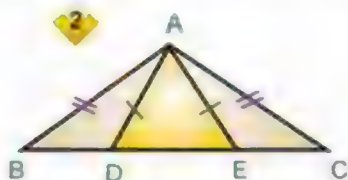
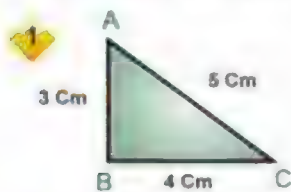






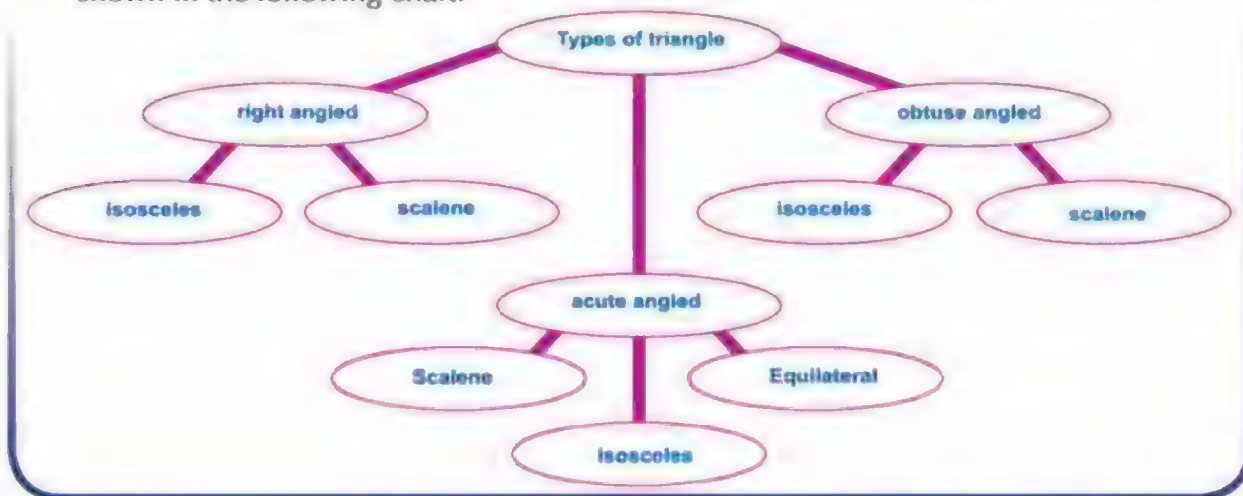
## Practice

In each of the following figures, state the isosceles triangles and define their bases, then notice the type of the two base angles and the vertex angle.



### Remark :

- Both of the base angles in the isosceles triangle are acute.
- The vertex angle in the isosceles triangle can be either acute, right or obtuse. So, the isosceles triangle can be either obtuse, right or acute angled triangle as shown in the following chart:



## UNIT FOUR

### Lesson Three

## The Isosceles Triangle Theorems

### Think and Discuss

#### You will learn how

- ✎ To define the relation between the base angles in the isosceles triangle.
- ✎ To define the relation among the measures of the angles in the equilateral triangle.
- ✎ To define the relation between two sides opposite to two equal angles in a triangle.
- ✎ To know that if the angles in a triangle are congruent, then the triangle is equilateral.

#### key terms

- ✎ The isosceles triangle.
- ✎ The base angles.

Is there a relation among the measures of the two base angles in the isosceles triangle?

to know that, let's conduct the next activity:




#### Activity

#### Using the compass

- 1 Draw several isosceles triangles as shown in the opposite figure Where  $AB = AC$ .



- 2  **Find** using a protractor, the measure of the two base angles  $\angle ABC$  and  $\angle ACB$
- 3 Write down the data you got in a table as follows, then compare the measures in each case.

Number of the triangle	$m(\angle ABC)$	$m(\angle ACB)$
1		
2		
3		

- 4 Keep your activity in the portfolio.



#### Theorem 1

(the isosceles triangle theorem) the base angles of the isosceles triangle are congruent.

**Given:** ABC is triangle in which  $\overline{AB} = \overline{AC}$

**R.T.P:**  $\angle B = \angle C$





**Construction :** draw  $\overline{AD} \perp \overline{BC}$

**Proof :** The two triangles ADB and ADC are right angled in which.

$$\begin{cases} \overline{AB} = \overline{AC} \\ \overline{AD} \end{cases}$$

(a given)

(a common side)

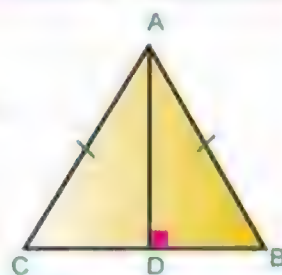
(a hypotenuse and a side)

$$\therefore \triangle ADB \cong \triangle ADC$$

from the congruency , we deduce that

$$\angle B = \angle C$$

Q.E.D.

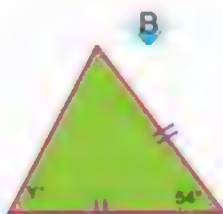


**practice**

- 1 In each of the following figures, find the value of the symbol that is used to measure the angle:



$$x =$$

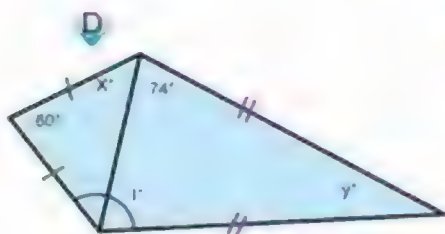


$$y =$$



$$N =$$

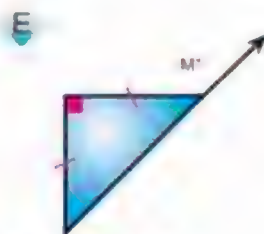
$$E =$$



$$x =$$

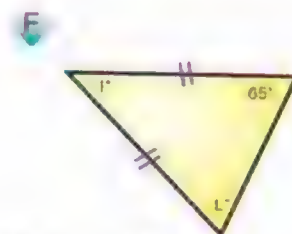
$$y =$$

$$l =$$



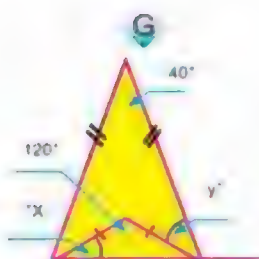
$$L =$$

$$M =$$



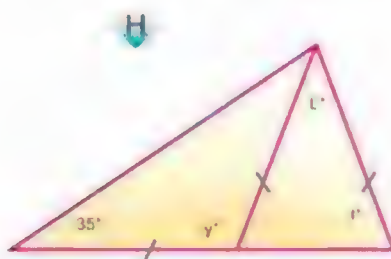
$$L =$$

$$l =$$



$$x =$$

$$y =$$



$$y =$$

$$L =$$

$$l =$$

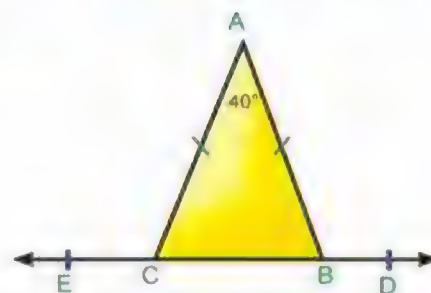


- 2 In the figure opposite, ABC is an isosceles triangle in which  $AB = AC$

$$m(\angle A) = 40^\circ, D \in \overrightarrow{CB}, E \in \overrightarrow{BC}.$$

First:  Find  $m(\angle ABC)$

Second:  Prove that  $\angle ABD = \angle ACE$



 **Think:** Are the supplementary angles to congruent angles congruent?

### Corollary

If the triangle is equilateral, then it is equiangular where each angle measure  $60^\circ$ .



### Example (1)

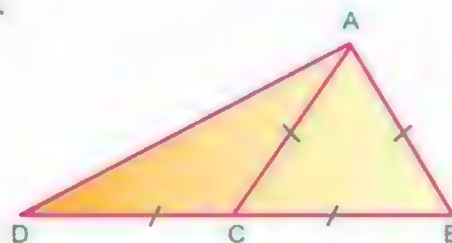
In the figure opposite: ABC is an equilateral triangle.

$D \in \overrightarrow{BC}$  such that  $BC = CD$

 prove that  $\overline{AB} = \overline{AD}$

**Given:**  $AB = BC = CA = CD, D \in \overrightarrow{BC}$

**R.T.P:** Prove that  $\overline{BA} = \overline{AD}$



**Proof :**  $\because \triangle ABC$  is an equilateral triangle.

$$\therefore m(\angle ACB) = m(\angle BAC) = m(\angle B) = 60^\circ \text{ (corollary)}$$

$$\because D \in \overrightarrow{BC}$$

$\therefore \angle BCA$  is an exterior angle of the  $\triangle ACD$

$$m(\angle BCA) = m(\angle CAD) + m(\angle CDA) = 60^\circ \quad (1)$$

In  $\triangle ACD$

$$\because CA = CD \quad \therefore m(\angle CAD) = m(\angle CDA) \quad (2)$$

from (1), (2) we deduce that:  $m(\angle CAD) = m(\angle CDA) = 30^\circ$





$$\therefore m(\angle BAD) = m(\angle BAC) + m(\angle CAD)$$

$$\therefore m(\angle BAD) = 60^\circ + 30^\circ = 90^\circ$$

$$\therefore \overline{BA} \perp \overline{AD} \quad \text{Q.E.D.}$$

**Remark:**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non - adjacent interior angles.

**Example (2)**

2 In the figure opposite:  $AB = AD$ ,  $BC = CD$

**Prove that**  $\angle ABC \cong \angle ADC$

**Given:**  $AB = AD$ ,  $BC = CD$

**R.T.P:** prove that  $\angle ABC \cong \angle ADC$

**Proof :** In  $\triangle ABD$

$$\therefore AB = AD$$

$$\therefore m(\angle ABD) = m(\angle ADB) \quad (1)$$

in  $\triangle CBD$

$$\therefore CB = CD$$

$$\therefore m(\angle CBD) = m(\angle CDB) \quad (2)$$

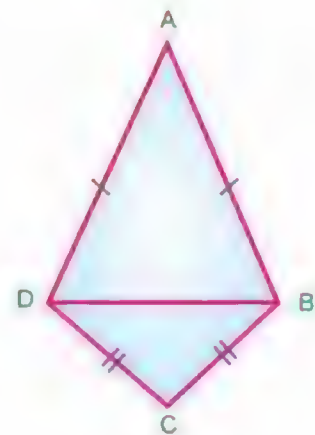
By adding (1) and (2) we deduce that

$$m(\angle ABD) + m(\angle CBD) = m(\angle ADB) + m(\angle CDB)$$

$$\therefore m(\angle ABC) = m(\angle ADC)$$

$$\angle ABC \cong \angle ADC$$

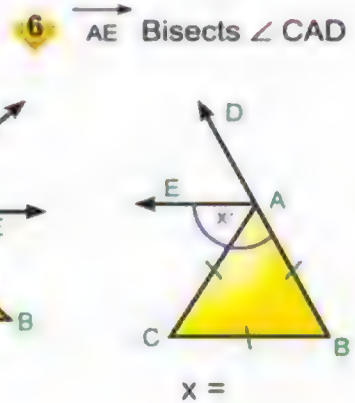
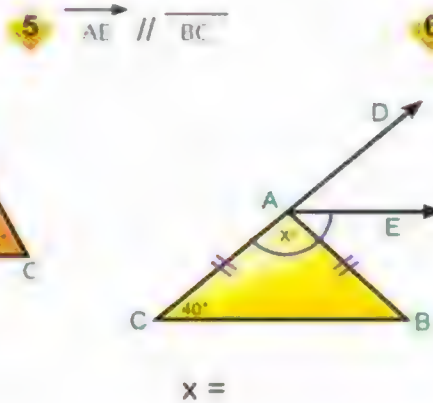
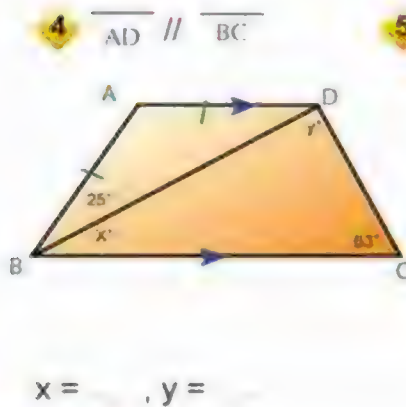
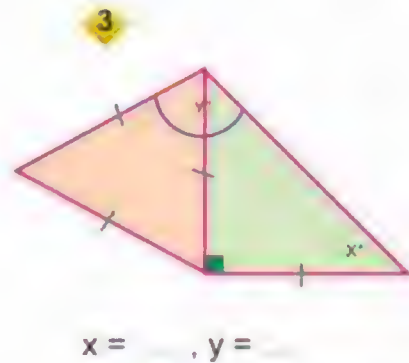
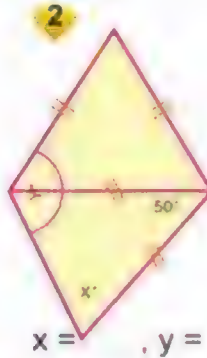
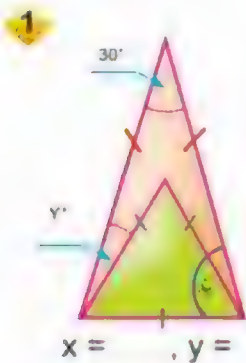
**Q.E.D.**





## practice

In each of the following figures, find the value of the symbol that is used to measure the angle:



## Activity

Draw the triangle ABC in which  $BC = 7$  cm,  $m(\angle B) = m(\angle C) = 50^\circ$ , then measure the lengths of both  $\overline{AB}$  and  $\overline{AC}$ . Repeat the activity using other measures for the length of  $\overline{BC}$  and the measures of angles B and C, then fill in the table:

Number of the triangle	BC	$m(\angle B)$	$m(\angle C)$	AB	AC
1	7cm	$50^\circ$	$50^\circ$		
2					
3					
4					

1 Are  $\overline{AB}$  and  $\overline{AC}$  equal in length?

2 Is  $\overline{AB} \cong \overline{AC}$ ?

3 How can you explain such corollaries geometrically?







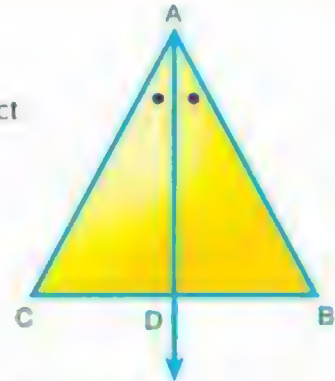
### Theorem (2)

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.

$$\triangle ABC, \angle B \cong \angle C$$

R.T.P: Prove that:  $\overline{AB} \cong \overline{AC}$

Construction : bisect  $\angle BAC$  with the bisector  $\overline{AD}$  to intersect  $\overline{BC}$  at D



Proof :  $\because \angle B \cong \angle C$

$$\therefore m(\angle B) = m(\angle C)$$

$\because \overline{AD}$  bisects  $\angle BAC$

$$\therefore m(\angle BAD) = m(\angle CAD)$$

$\because$  the sum of the measures of interior angles of a triangle is  $= 180^\circ$

$$\therefore m(\angle ADB) = m(\angle ADC)$$

$\therefore$  In the two triangles ADB, ADC

$\overline{AD}$  is a common side

$$m(\angle BAD) = m(\angle CAD)$$

$$m(\angle ADB) = m(\angle ADC)$$

$$\therefore \triangle ADB \cong \triangle ADC$$

Form the congruency, we deduce that  $\overline{AB} \cong \overline{AC}$

Therefore,  $\triangle ABC$  is an isosceles triangle



### Corollary

If the angles of a triangle are congruent, then the triangle is equilateral.

In the figure opposite ABC is an isosceles triangle in which:

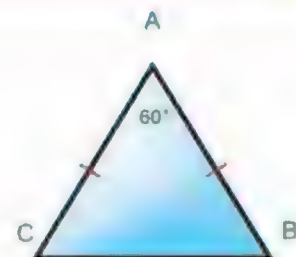
$$AB = AC, m(\angle BAC) = 60^\circ$$



Complete  $m(\angle ABC) = m(\angle ACB) = \dots\dots\dots$

i.e:  $\angle \dots \cong \angle \dots \cong \angle \dots$

$\therefore \triangle ABC$  is  $\dots\dots\dots$  triangle



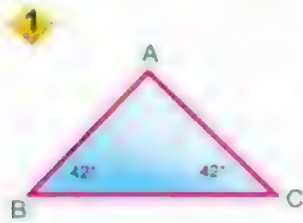
**Remark:**

In an isosceles triangle, If any angle has a measure of  $60^\circ$ , then the triangle is an equilateral triangle.

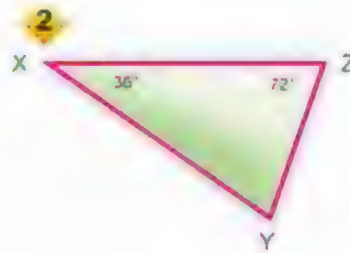


**practice**

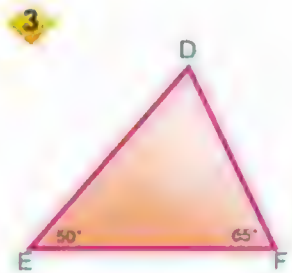
In each of the following figures, define the triangle's sides that are equal in length as shown in example 1 :



$AB = AC$



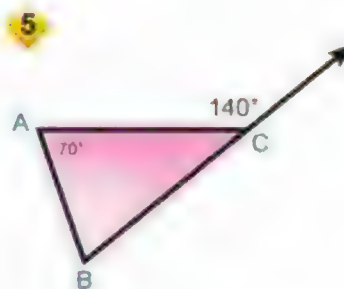
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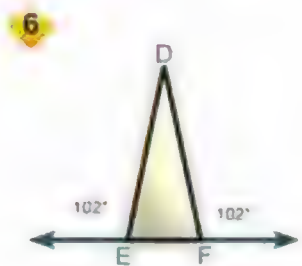
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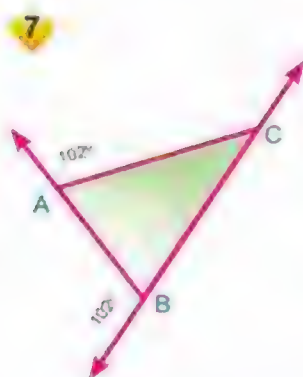
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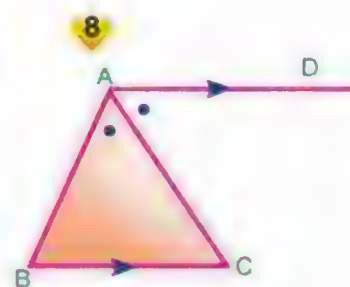
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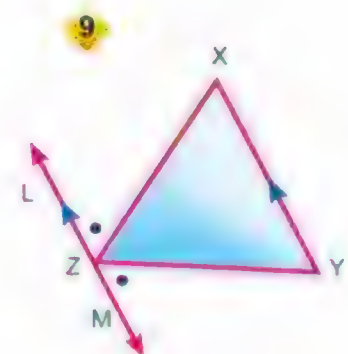
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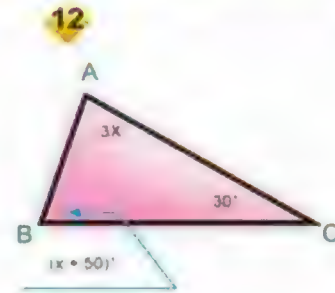
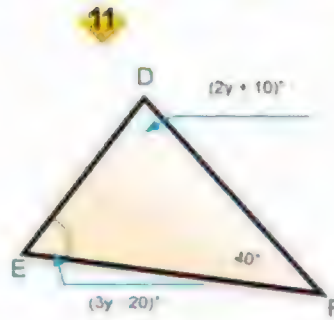
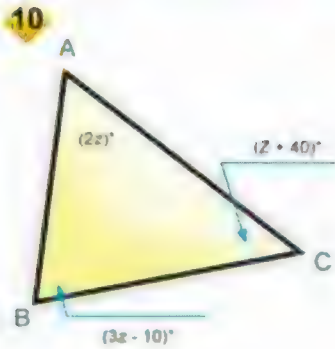
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### Examples

1 In the figure opposite:  $ABC$  is a triangle in which  $AB = AC$ ,  $\overline{XY} \parallel \overline{BC}$

**prove that**  $\triangle AXY$  is an isosceles triangle

**Given:**  $AB = CA$ ,  $\overline{XY} \parallel \overline{BC}$ .

**Required:** prove that  $AX = AY$

**Proof:** In  $\triangle ABC$   $\therefore AB = AC$

$$\therefore m(\angle ABC) = m(\angle ACB) \quad (1)$$

$\therefore \overline{XY} \parallel \overline{BC}$ ,  $\overleftrightarrow{AB}$  a transversal

$$\therefore m(\angle AXY) = m(\angle ABC) \text{ correspondingly} \quad (2)$$

The same  $\overline{XY} \parallel \overline{BC}$ ,  $\overleftrightarrow{AC}$  a transversal

$$\therefore m(\angle AYX) = m(\angle ACB) \text{ correspondingly} \quad (3)$$

from (1), (2), (3) we deduce that :

$$m(\angle AXY) = m(\angle AYX)$$

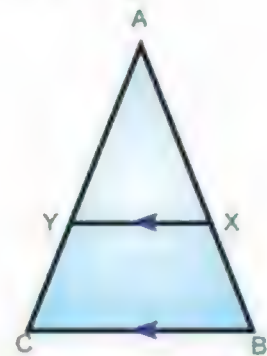
In  $\triangle AXY$

$$\therefore m(\angle AXY) = m(\angle AYX)$$

$$\therefore AX = AY$$

i.e. the triangle  $AXY$  is an isosceles triangle

Q.E.D



**Think :** Can we deduce that  $XB = YC$  ? Explain your answer,



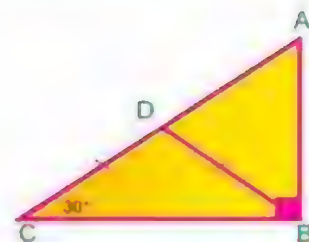
2 In the figure opposite :

ABC is a right angled triangle at B,  $m(\angle C) = 30^\circ$ ,

$D \in \overline{AC}$  where  $DB = DC$



**prove that**  $\triangle ABD$  is an equilateral triangle.



**Given:**  $m(\angle ABC) = 90^\circ$ ,  $m(\angle C) = 30^\circ$ ,  $DB = DC$

**R.T.P:** prove that  $AB = BD = AD$

**proof:** In  $\triangle DBC \because DB = DC$

$$\therefore m(\angle DBC) = m(\angle C) = 30^\circ$$

$$\text{in } \triangle ABC \because m(\angle ABC) = 90^\circ, m(\angle DBC) = 30^\circ$$

$$\therefore m(\angle BAD) = 90 - 30 = 60^\circ \quad (1)$$

$\because \angle ADB$  is an exterior angle of  $\triangle BDC$

$$\therefore m(\angle ADB) = m(\angle DBC) + m(\angle DCB)$$

$$m(\angle ADB) = 30^\circ + 30^\circ = 60^\circ \quad (2)$$

In  $\triangle ABD \because$  the sum of the measures of the interior angles of a triangle =  $180^\circ$

$$\therefore m(\angle ABD) = 180^\circ - (60^\circ + 60^\circ) = 60^\circ \quad (3)$$

$$\text{from (1), (2), (3)} \quad \therefore m(\angle ABD) = m(\angle ADB) = m(\angle A)$$

$$\text{i.e. } \angle ABD \cong \angle ADB \cong \angle A$$

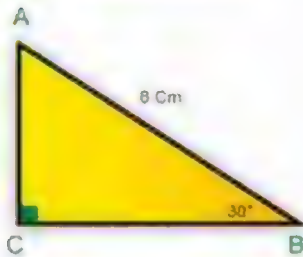
$$\therefore \text{the triangle ABD is equilateral} \quad \text{i.e. } AB = BD = AD$$





## Exercises (4-3)

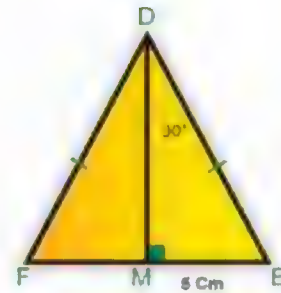
1 Complete:



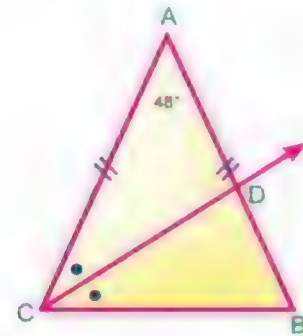
AC = \_\_\_\_\_



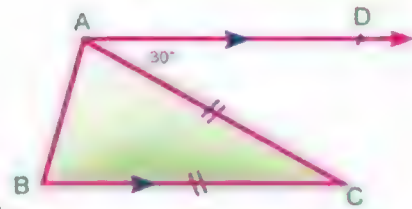
XZ = \_\_\_\_\_ cm

DE = \_\_\_\_\_ cm,  $m(\angle E) = \dots^\circ$ EF = \_\_\_\_\_ cm,  $m(\angle MDF) = \dots^\circ$ 

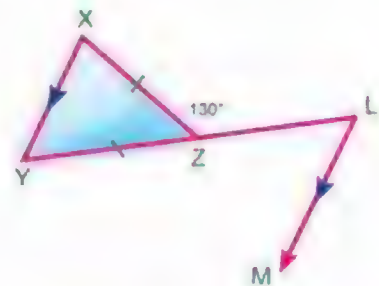
2 In the figure opposite :

 $AB = AC$ ,  $m(\angle BAC) = 48^\circ$  $\overrightarrow{CD}$  bisects  $\angle BCA$  and intersects  $\overline{AB}$  at D Find  $m(\angle B)$  et  $m(\angle BCD)$ 

3 In the figure opposite :

ABC is a triangle in which  $AC = BC$ . $\overrightarrow{AD} \parallel \overrightarrow{BC}$ ,  $m(\angle DAC) = 30^\circ$  Find The measures of the angles in  $\triangle ABC$ 

4 In the figure opposite :

 $Z \in \overline{LY}$ ,  $XZ = YZ$  $m(\angle LZX) = 130^\circ$ ,  $\overrightarrow{LM} \parallel \overrightarrow{XY}$  Find  $m(\angle MLY)$ 

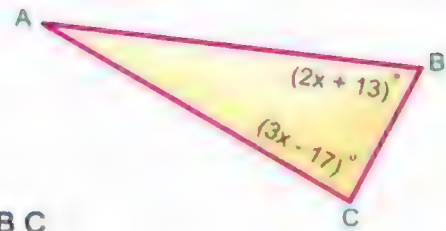
5 In the figure opposite :

$$AB = AC, m(\angle B) = (2x + 13)^\circ$$

$$m(\angle C) = (3x - 17)^\circ$$



**Find** the measures of the angles of  $\triangle ABC$



6 In the figure opposite:

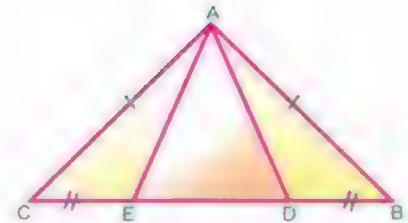
$ABC$  is an isosceles triangle in which  $AB = AC$ ,

$D \in \overline{BC}$ ,  $E \in \overline{BC}$  such that  $BD = EC$



**Prove that** First:  $\triangle ADE$  is an isosceles triangle.

Second:  $\angle ADE \cong \angle AED$



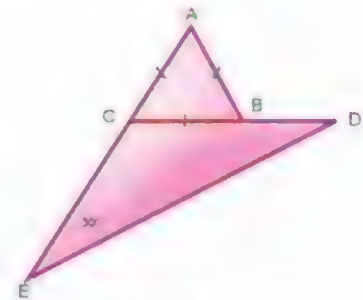
7 In the figure opposite :  $ABC$  is an equilateral triangle.

$E \in \overline{AC}$ ,  $D \in \overline{CB}$ ,

$$m(\angle DEC) = 30^\circ$$



**Prove that**  $\triangle DCE$  is an isosceles triangle.



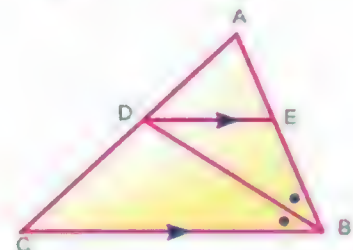
8 In the figure opposite :

$\overrightarrow{BD}$  bisects  $\angle ABC$  and intersects  $\overline{AC}$  at  $D$ ,

$\overline{DE} \parallel \overline{BC}$  where  $E \in \overline{AB}$ .



**Prove that**  $\triangle EBD$  is an isosceles triangle.



9  $ABC$  is a triangle in which  $D \in \overline{AB}$ ,

$E \in \overline{BC}$  such that  $BD = BE$ , So if  $\overline{DE} \parallel \overline{AC}$



**Prove that**  $AB = BC$

10  $ABC$  is a triangle in which  $AB = AC$ ,

$\overrightarrow{BD}$  bisects  $\angle ABC$ ,  $\overrightarrow{CD}$  bisects  $\angle ACB$ ,  $\overrightarrow{BD} \cap \overrightarrow{CD} = \{D\}$



**Prove that**  $\triangle DBC$  is an isosceles triangle.



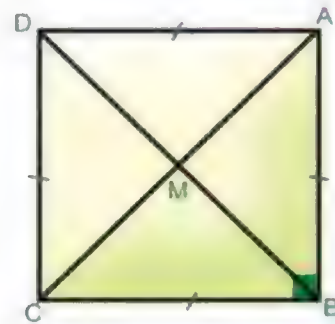


- 11 ABCD is a square in which its diagonals  $\overline{AC}$  ,  $\overline{BD}$  intersect at M



**Complete and discuss.**

- A In  $\triangle ABC$  ,  $m(\angle ABC) = \dots\dots\dots^\circ$   
 $\therefore AB = BC$   
 $\therefore m(\angle BAC) = m(\angle BCA) = \dots\dots\dots^\circ$
- B  $\therefore m(\angle BAD) = 90^\circ$   $\therefore m(\angle DAC) = \dots\dots\dots^\circ$   
 $\therefore m(\angle BCD) = 90^\circ$   $\therefore m(\angle ACD) = \dots\dots\dots^\circ$
- C Does the diagonal  $\overline{AC}$  bisect  $\angle A$ ?
- D Does the diagonal  $\overline{BD}$  bisect both  $\angle B$  and  $\angle D$ ?
- E Is  $\triangle MAD$  an isosceles triangle? why?
- F State those isosceles triangles whose vertices are point M.
- G Is M the midpoint of  $\overline{AC}$  and  $\overline{BD}$ ?
- H Is  $\overline{AC} = \overline{BD}$ ?
- ↓ From the previous clauses, deduce the properties of the square and keep them in



# UNIT FOUR

## Lesson Four

### Corollaries of isosceles triangle theorems

#### Think and Discuss

#### You will learn how

- The corollaries on the theorems of isosceles triangles.

#### key terms

- The isosceles triangle
- The bisector of a vertex angle
- The bisector of a triangle base.
- The axis of symmetry for a line segment..

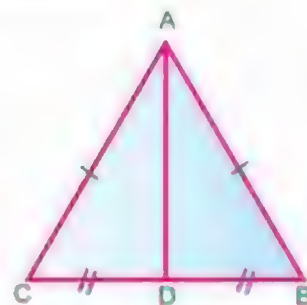
#### Corollary (1)

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the figure opposite:

In  $\triangle ABC$ ,  $AB = AC$ ,  $\overline{AD}$  is a median

then:  $\overrightarrow{AD}$  bisects  $\angle BAC$ .  $\overline{AD} \perp \overline{BC}$



**Remark:**  $\triangle ADB \equiv \triangle ADC$ . Why?

#### Corollary (2)

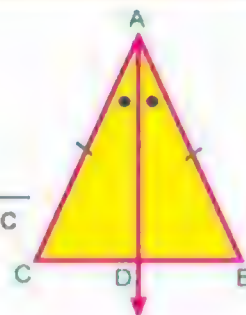
The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the figure opposite:

In  $\triangle ABC$ ,  $AB = AC$ ,

$\overrightarrow{AD}$  bisects  $\angle BAC$

then D is a midpoint of  $\overline{BC}$  and  $\overline{AD} \perp \overline{BC}$



**Remark:**  $\triangle ADB \equiv \triangle ADC$ . why?







### Corollary (3)

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the figure opposite :

In  $\triangle ABC$ ,  $AB = AC$ ,  $\overline{AD} \perp \overline{BC}$

then D bisects  $\overline{BC}$ ,  $m(\angle BAD) = m(\angle CAD)$

**Remark :**

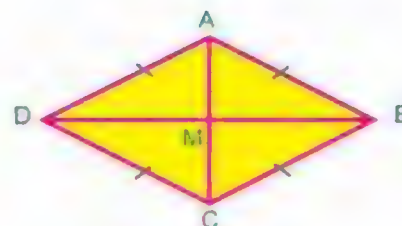
$\triangle ADB \cong \triangle ADC$ . why?



**Think**

In the figure opposite :

ABCD is a quadrilateral in which all sides are equal in length, this figure is called rhombus, its diagonals are  $\overline{AC}$  and  $\overline{BD}$ . they intersect at point M



**Remark :**

$\triangle ABD \cong \triangle CBD$ . why?

$\therefore m(\angle ABD) = m(\angle CBD)$

in  $\triangle ABC$ ,  $AB = BC$ ,  $\overline{BM}$  bisects  $\angle ABC$

$\therefore \overline{BM} \perp \dots\dots\dots$ , M is the midpoint of  $\overline{AC}$

in  $\triangle BAD$ ,  $AB = AD$ ,  $\overline{AM} \perp \overline{BD}$

$\therefore \overline{AM}$  bisects  $\angle \dots\dots\dots$ , M is the midpoint of  $\overline{BD}$

Are the two diagonals of the rhombus perpendicular?

Do the two diagonals of the rhombus bisect each other?

Does the diagonal of the rhombus bisect the vertex angles which it connects?

Write down your answer.



## Axes of symmetry

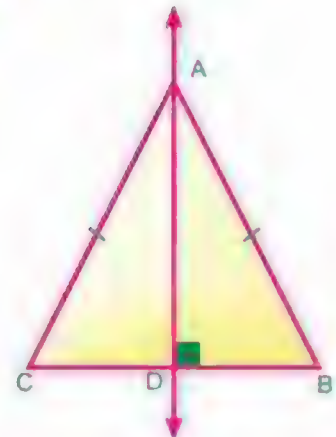
### First: axes of symmetry in the isosceles triangle:

The axis of symmetry of the isosceles triangle is the straight line drawn from the vertex angle perpendicular to its base.

In the figure opposite:

$\triangle ABC$  in which  $AB = AC$ ,  $AD \perp BC$

then  $\overleftrightarrow{AD}$  is the axis of symmetry in the isosceles triangle  $ABC$ .



### Discuss:

Does the isosceles triangle has more than one axis of symmetry?

- How many axes of symmetry are there in the equilateral triangle?
- Are there any axes of symmetry in the scalene triangle?

### Second: Axis of symmetry of a line segment :

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment in brief it is known as the axis of a line segment.

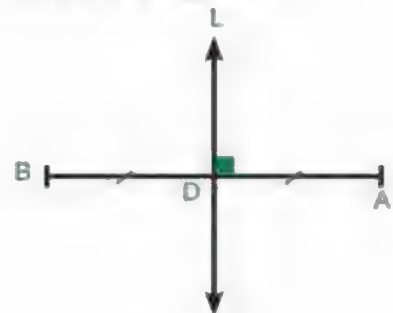
In the figure opposite:

If D the midpoint of  $\overline{AB}$  and

The straight line  $L \perp AB$

Where  $D \in L$ , then the straight line L

Is the axis of  $\overline{AB}$

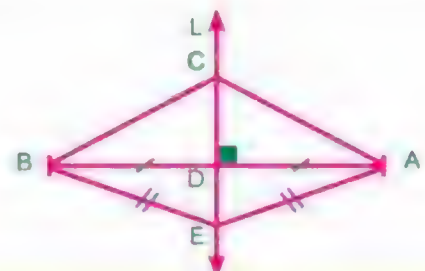


### Important property

Any point at the axis of symmetry of a line segment is at equal distances from its end points.

### Remark:

- ① If  $C \in L$  then  $AC = BC$
- ② If  $EA = EB$  then  $E \in L$ . why?





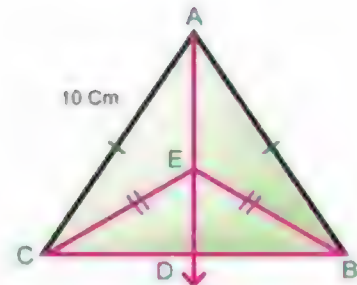
### Examples

#### 1 In the figure opposite

$$AB = AC = 10 \text{ cm}, EB = EC$$

$$\overrightarrow{AE} \cap \overrightarrow{BC} = \{D\}$$

If  $BC = 6 \text{ cm}$ , find the length of  $CD$  and  $AD$



**Given :**  $AB = AC, EB = EC$

**R.T.P :** Find  $CD$  and  $AD$

**Proof :**  $\because AB = AC \therefore A$  is on the axis of  $\overline{BC}$   
 $\because EB = EC \therefore E$  is on the axis of  $\overline{BC}$   
 $\therefore \overleftrightarrow{AE}$  is the axis of  $\overline{BC}$   
 $D$  is the midpoint of  $\overline{BC}, AD \perp BC$   
 $\because D$  is the midpoint of  $\overline{BC}, BC = 6 \text{ cm} \therefore CD = 3 \text{ cm}$   
 $\because AD \perp BC$   
 $\therefore$  In  $\triangle ADC$  that is right angled triangle at  $D$   
 $(AD)^2 = (AC)^2 - (CD)^2$   
 $(AD)^2 = 100 - 9$   
 $\therefore AD = \sqrt{91} \text{ cm}$

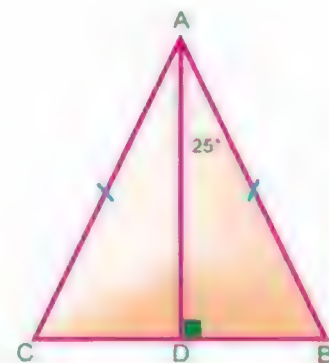
#### 2 In the figure opposite

$ABC$  is a triangle in which  $AB = AC$ ,

$AD \perp BC, m(\angle BAD) = 25^\circ$ ,

$BC = 4 \text{ cm}$ . Find:

- A  $m(\angle DAC)$       B the length of  $\overline{DC}$



#### Solution

**Given:**  $AB = AC,$

$AD \perp BC, m(\angle BAD) = 25^\circ, BC = 4 \text{ cm}$

**R.T.P:**  $m(\angle DAC)$ , and the length of  $\overline{DC}$ .





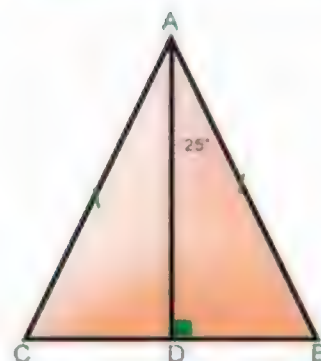
**Proof** : in  $\triangle ABC$

$$\therefore AB = AC, \overline{AD} \perp \overline{BC}$$

$\therefore \overline{AD}$  bisects both of the base  $\overline{BC}$  and  $\angle BAC$

$$\therefore m(\angle DAC) = m(\angle DAB) = 25^\circ,$$

$$DC = \frac{1}{2} BC = \frac{4}{2} = 2 \text{ cm.}$$



### practice

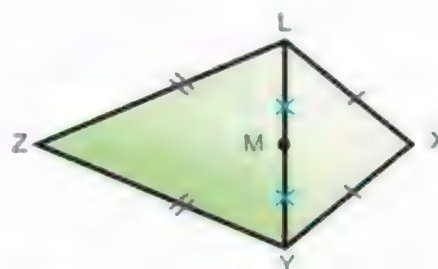
1

In the figure opposite

$$xy = xL, Zy = ZL, LM = YM$$



**Prove that** X, M and Z are on the same straight line



2

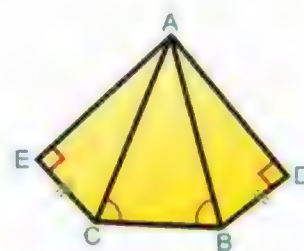
In the figure opposite:

$$BD = CE$$

$$m(\angle ABC) = m(\angle ACB)$$

$$m(\angle D) = m(\angle E) = 90^\circ$$

**Prove that** :  $m(\angle DAB) = m(\angle CAE)$



3

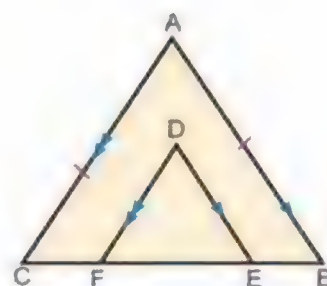
In the figure opposite:

$$AB = AC, \overline{DE} \parallel \overline{AB}$$


$$\overline{DF} \parallel \overline{AC}$$

**Prove that:**  $DE = DF$

**Second:**  $m(\angle BAC) = m(\angle EDF)$



## Exercises (4-4)

1  **Complete** to get a correct statement :

- A The vertex angle bisector in the isosceles triangle bisects the base and is
- B The number of symmetrical axes in the equilateral triangle is
- C Any point at the axis of a line segment symmetry is at two equal distances from
- D If the measurement of an angle in the isosceles triangle is  $100^\circ$ , then the measurement of an angle of the other two = .....  $^\circ$ .

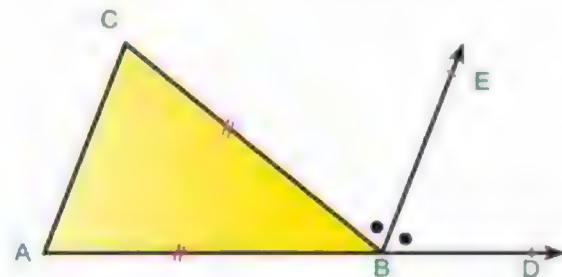
2  **Choose the correct answer:**

- A The number of axes of symmetry in the isosceles triangle = .....  
(0 , 1 , 2 , 3)
- B The triangle whose sides lengths are 2cm, (X + 3) and 5cm becomes an isosceles triangle when X = ..... cm.  
(1 , 2 , 3 , 4)
- C The intersecting point of the medians of a triangle divides each other from the direction of the base in a ratio .  
(1 : 2 , 2 : 1 , 1 : 3 , 2 : 3)

3  **In the figure opposite:**

$AB = BC$ ,  $\overrightarrow{BE}$  bisects  $\angle CBD$   
 $D \in \overrightarrow{AB}$

Prove that:  $\overrightarrow{BE} \parallel \overrightarrow{AC}$

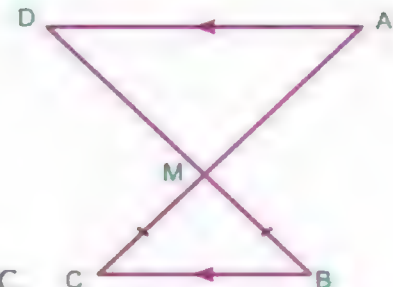


4  **In the figure opposite:**

$\overline{AC} \cap \overline{BD} = \{M\}$   
 $\overline{AD} \parallel \overline{BC}$  ,  $MB = MC$

Prove that :

- (1)  $\triangle AMD$  is an isosceles triangle
- (2) The axis of symmetry of  $\triangle AMD$  is the same of  $\triangle BMC$ .





## General Exercises



### 1 In the figure opposite

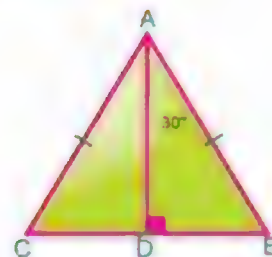
$AB = AC$ ,  $BC = 10\text{cm}$ ,

$m(\angle BAD) = 30^\circ$ ,  $\overline{AD} \perp \overline{BC}$

**First:** Find the length of  $\overline{BD}$ ,  $\overline{AD}$ .

**Second:** How many axes of symmetry are there at  $\triangle ABC$ ?

**Third:** What is the area of  $\triangle ABC$ ?



### 2 In the figure opposite

$AB = AC$ ,  $D \in \overrightarrow{AB}$ ,  $E \in \overrightarrow{AC}$

$\overrightarrow{BF}$  bisects  $\angle DBC$ ,

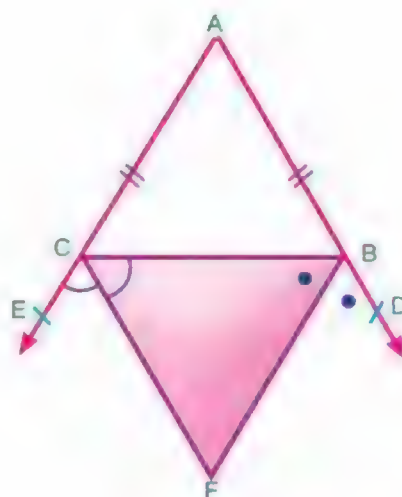
$\overrightarrow{CF}$  bisects  $\angle BCE$



**Prove that**

**First:**  $\triangle BFC$  is an isosceles triangle

**Second:**  $\overleftrightarrow{AF}$  is the axis of symmetry of  $\overline{BC}$



### 3 In the figure opposite

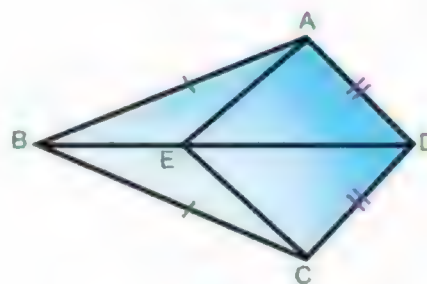
$AB = CB$ ,  $AD = CD$ ,  $E \in \overline{BD}$



**Prove that**

$\overrightarrow{BD}$  bisects  $\angle ADC$

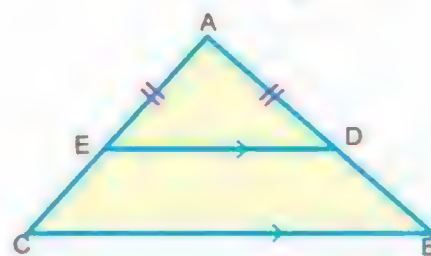
$\overrightarrow{DE}$  bisects  $\angle ABC$



### 4 In the figure opposite

$\overline{DE} \parallel \overline{BC}$ ,  $AD = AE$

prove that:  $AB = AC$ .



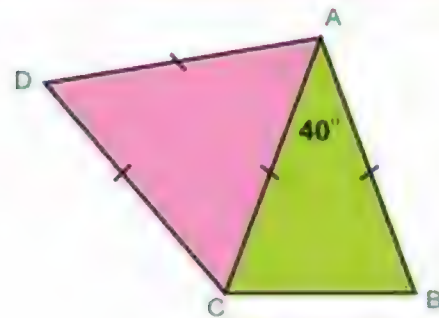


5 In the figure opposite

$$AB = AC = AD = CD$$

$$m(\angle BAC) = 40$$

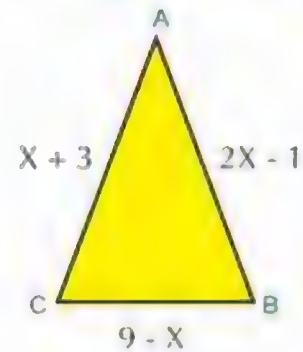
Find:  $m(\angle BCD)$



6 In the figure opposite

ABC is a triangle which  $m(\angle B) = m(\angle C)$

Find: The perimeter of in the triangle



7 In the figure opposite

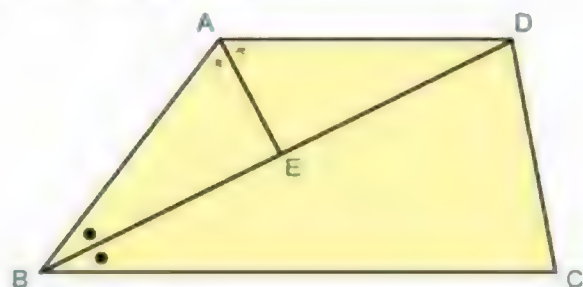
ABCD is a quadrilateral in which  $\overline{AD} \parallel \overline{BC}$ ,  $\overline{BD}$  bisects  $\angle ABC$ ,  $\overline{AE}$  bisects  $\angle BAD$

Prove that

A  $AB = AD$

B  $\overline{AE} \perp \overline{BD}$

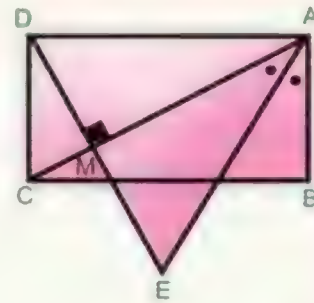
C  $BE = ED$



## Activity

- 1 Using a ruler and a compass, draw the acute angle  $ABC$ . At the opposite side from  $\overrightarrow{BA}$ , draw  $\overrightarrow{AE} \parallel \overrightarrow{BC}$

- 2 In the opposite figure  $ABCD$  is a rectangle in which  $\overline{AC}$  is a diagonal,  $\overrightarrow{AE}$  bisects  $\angle BAC$ ,  $\overline{DE} \perp \overline{AC}$  where  $\overrightarrow{AE} \cap \overrightarrow{DE} = \{E\}$   
 $\overline{AC} \cap \overline{DE} = \{M\}$



**Prove that**  $DA = DE$ .



## Unit test

1



**Complete** the following to get a correct statement :

- A The base angles in an isosceles triangle are .....
- B The median that is drawn from the vertex of an isosceles triangle is and
- C In the triangle ABC,  $AB = AC$ ,  $m(\angle A) = 70^\circ$ , so  $m(\angle C) =$
- D The number of the axes of symmetry in an equilateral triangle =
- E The measure of the exterior angle in an equilateral triangle =
- F The straight line perpendicular to the midpoint of a line segment is called

2

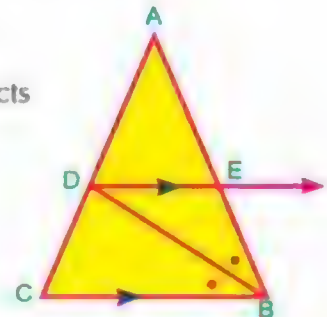
**In the figure opposite :**

ABC is a triangle in which  $\overrightarrow{BD}$  bisects  $\angle ABC$  and intersects

$\overline{AC}$  at D,  $\overrightarrow{DE} \parallel \overline{CB}$

$\overrightarrow{DE} \cap \overline{AB} = \{E\}$

**Prove that**  $BE = ED$



3

**In the figure opposite** ABC is a triangle in which

$AB = AC$ ,  $\overrightarrow{AE}$  bisects  $\angle BAC$ ,

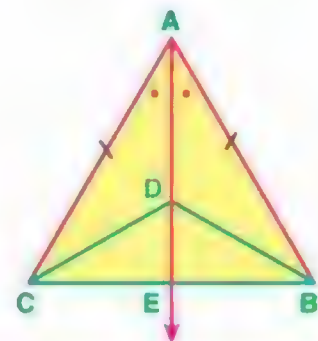
$\overline{AE} \cap \overline{BC} = \{E\}$

$D \in \overline{AE}$ .

**Prove that**

A  $BE = \frac{1}{2} BC$

B  $BD = CD$

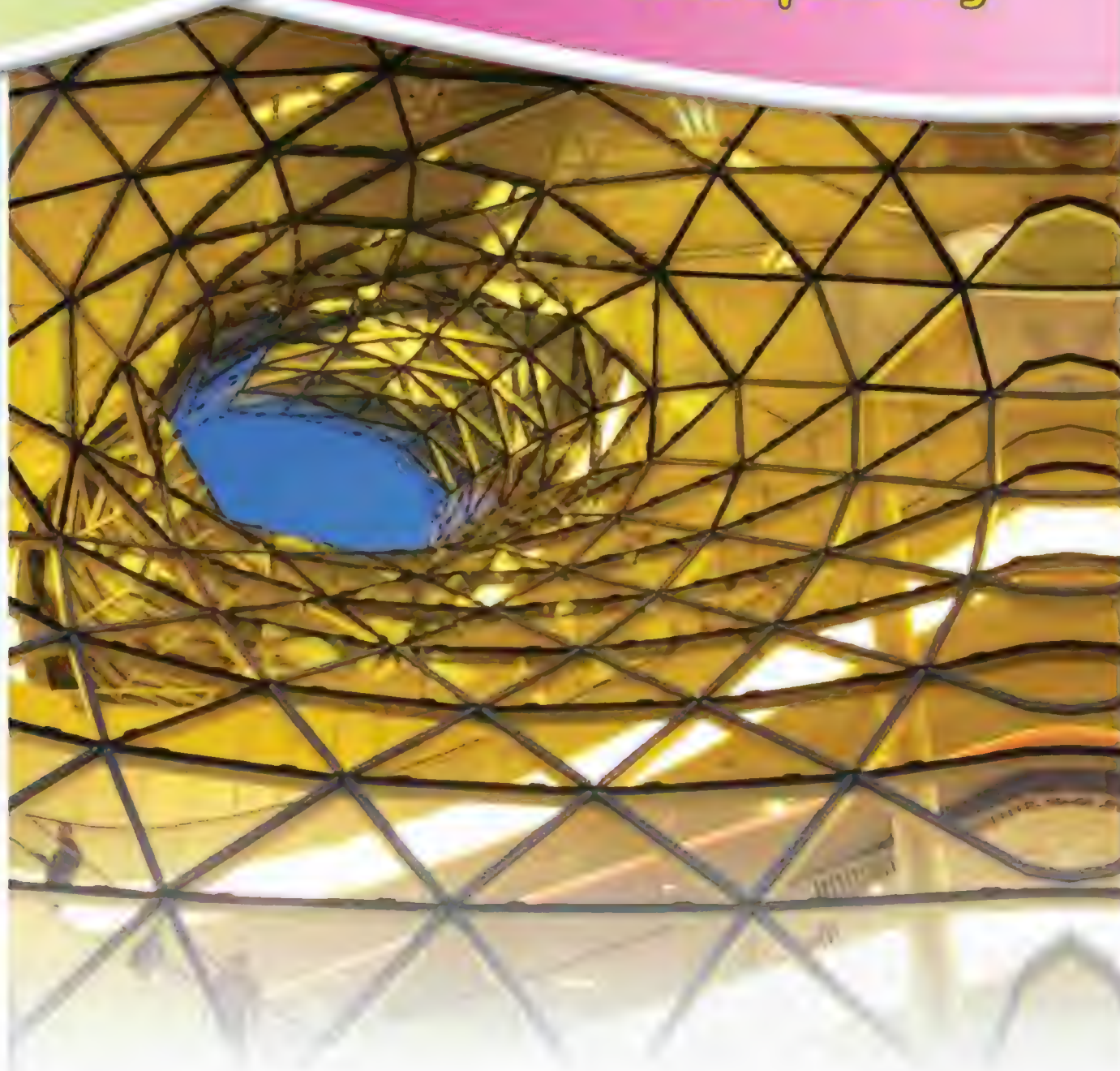




## UNIT FIVE

### 5

# Inequality



# UNIT FIVE

## Lesson One

## Inequality

### Think and Discuss

#### You will learn how

- To define the concept of inequality.
- To define axioms of inequality.

#### key terms

- inequality.
- axioms.
- greater than  $>$ .
- Less than  $<$ .
- equal to

#### The concept of inequality:

- 1 Do all the students in your class have the same height?
- 2 Are there any differences among the measures of acute, right and obtuse angles?

#### What does this difference mean?

#### Remark

An Inequality means that there is a difference in the heights of the students and in the measures of the angles. This difference is represented by the relation of inequality which is used to compare two different numbers.



#### Examples

- 1 If:  $\angle ABC$  is an acute angle then:  $m(\angle ABC) < 90^\circ$

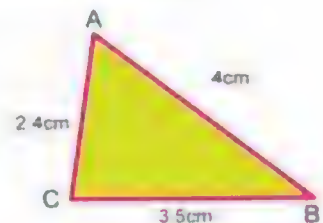
- 2 In the figure opposite,  $ABC$  is a triangle in which:

$AB = 4\text{cm}$ ,  $BC = 3.5\text{cm}$ ,

$AC = 2.4\text{cm}$

then:  $AB > BC$ ,  $BC > AC$

i.e.  $AB > BC > AC$







**practice:**

In the figure opposite, find:  $m(\angle ACB)$ ,  $m(\angle ACD)$

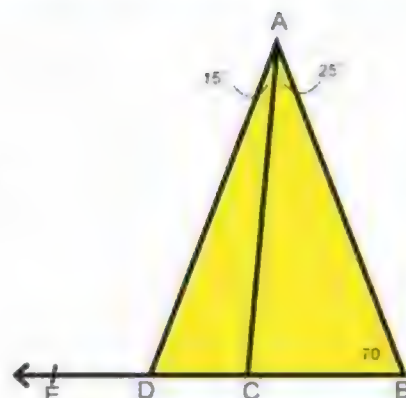
and  $m(\angle ADE)$  then complete by using  $>$  or  $<$ :

$m(\angle ADE)$  .....  $m(\angle CAD)$

$m(\angle ADC)$  .....  $m(\angle ACB)$

$m(\angle ACD)$  .....  $m(\angle ABC)$

$m(\angle ACD)$  .....  $m(\angle ADE)$



**Remark:**

All the previous relations are called inequalities.

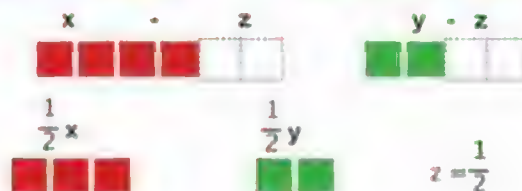
### Axioms of inequality

For any given three numbers  $x$ ,  $y$  and  $z$ :

- 1 If:  $x > y$   
then:  $x + z > y + z$



- 2 If:  $x > y$   
then:  $x - z > y - z$



- 3 If:  $x > y$ ,  $z$  is a positive number  
then:  $xz > yz$



- 4 If:  $x > y$ ,  $y > z$   
then:  $x > z$



- 5 If:  $x > y$ ,  $A > B$   
then:  $x + A > y + B$





## Remember

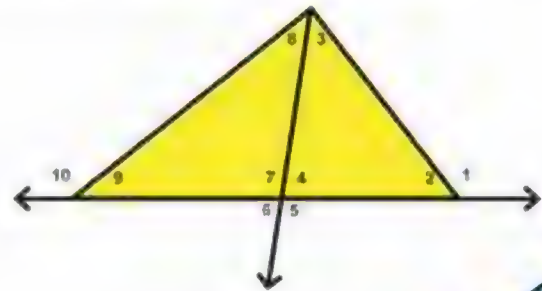
The measure of any exterior angle of a triangle is greater than the measure of any interior angle except for the adjacent angle.



## practice

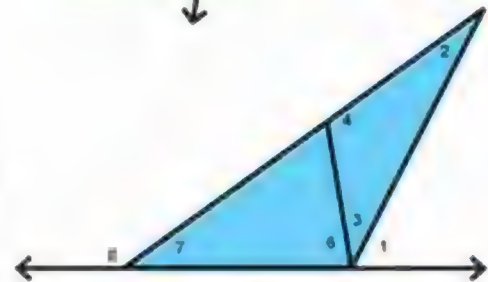
- 1 In the figure opposite: which of the following angles has the greatest measure?

- A  $\angle 1$  ,  $\angle 3$  ,  $\angle 4$
- B  $\angle 4$  ,  $\angle 8$  ,  $\angle 9$
- C  $\angle 2$  ,  $\angle 3$  ,  $\angle 7$
- D  $\angle 7$  ,  $\angle 8$  ,  $\angle 10$

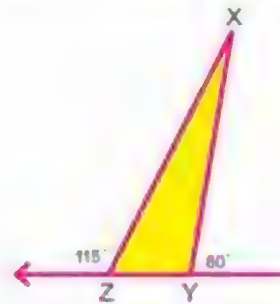
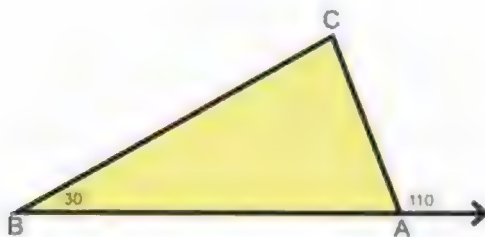


- 2 In the figure opposite, find:

- A All angles of measures less than  $m(\angle 1)$
- B All angles of measures greater than  $m(\angle 6)$
- C All angles of measures less than  $m(\angle 4)$



- 3 Order the measures of the angles in the triangle ABC in an ascending order and the measures of the angles in the triangle XYZ in a descending order.



$$m(\angle \dots) < m(\angle \dots) < m(\angle \dots)$$

$$m(\angle \dots) > m(\angle \dots) > m(\angle \dots)$$

- 4 In the figure opposite:  $C \in \overleftrightarrow{AB}$  ,  $D \in \overleftrightarrow{AB}$

If:  $AB > CD$

then:  $AC \dots BD$





### Example

In the figure opposite :

$$m(\angle ACB) > m(\angle ABC), DB = DC$$

**Prove that :**  $m(\angle ACD) > m(\angle ABD)$

**Given:**  $m(\angle ACB) > m(\angle ABC), DB = DC$

**Required to prove:**  $m(\angle ACD) > m(\angle ABD)$

**R.T.P:**  $\because DB = DC$

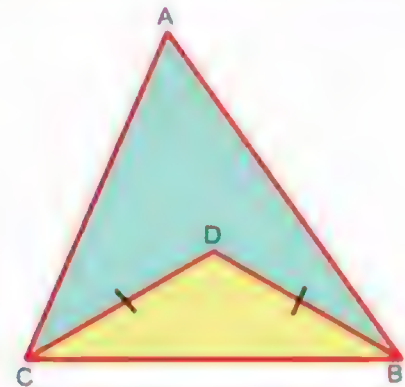
$$\therefore m(\angle DCB) = m(\angle DBC) \quad (1)$$

$$\therefore m(\angle ACB) > m(\angle ABC) \quad (2)$$

$\therefore$  By subtracting (1) from (2), we get:

$$m(\angle ACB) - m(\angle DCB) > m(\angle ABC) - m(\angle DBC)$$

$$\therefore m(\angle ACD) > m(\angle ABD) \quad \text{Q.E.D}$$



## Exercises (5-1)

1 In the figure opposite :

ABC is a triangle in which  $AC > AB$ ,  $x \in \overline{AB}$

$y \in \overline{AC}$ , where  $m(\angle AXY) = m(\angle AYX)$

**Prove that:**  $YC > XB$

2 In the figure opposite :  $\overline{AB} \parallel \overline{CD}$ ,

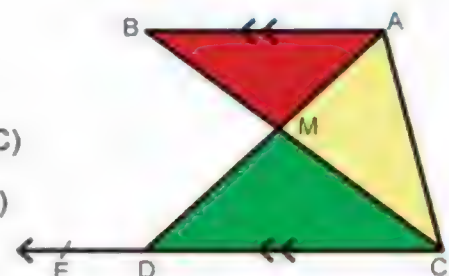
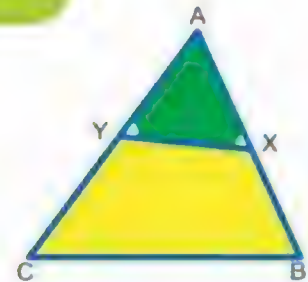
$\overline{AD} \cap \overline{CB} = \{M\}$ ,  $E \in \overline{CD}$ ,  $E \notin \overline{CD}$

**Prove that:** A  $m(\angle ACD) > m(\angle ABC)$

B  $m(\angle ADE) > m(\angle ABC)$

3 M is a point in a triangle ABC,

**Prove that:**  $m(\angle AMB) > m(\angle ACB)$



# UNIT FIVE

## Lesson Two

### Comparing the measures of the angles of a triangle

#### Think and Discuss

#### You will learn how

- To compare the measures of angles in a triangle.

#### key terms

- Angle.
- Measure of an angle.
- The greatest angle in a triangle.
- The smallest angle in a triangle.
- The largest side of a triangle.
- The smallest side of a triangle...



#### Activity

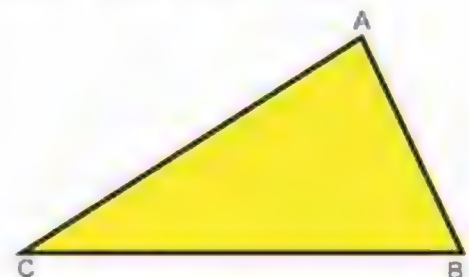
- 1** In the figure opposite: ABC is an isosceles triangle in which  $AB = AC$

- Fold the triangle to make the vertex B congruent to vertex C. What do you observe regarding to the measures of the angles B, C which are opposite to the two equal sides  $AC$ ,  $AB$ ?



- Fold the triangle to make the vertices A, C congruent. what do you observe regarding to the measures of the two angles opposite to the two unequal sides  $BC$ ,  $AB$ ?
- Does the difference in the lengths of the two sides in a triangle lead to a difference in the measures of their two opposite angles?

- 2** Draw the scalene triangle. ABC Flip the triangle to make the vertex A coincide the vertex B. What do you observe regarding to the measures of the two angles A, and B that are opposite to the two unequal sides,  $BC$ ,  $AC$ .



- Repeat the previous steps to make the vertex B coincide vertex C. what do you observe?





Are there any equal angles in measures in that triangle?

**Notice that:**

In a triangle, if the sides are unequal in length, the measures of the opposite angles are unequal.



### Activity

Draw the scalene triangle ABC, then measure the lengths of its 3 sides and the measures of the opposite angles, then complete the following table::

Lengths of sides	Measures of the opposite angles
AB =      cm	$m(\hat{C}) = \quad \cdot$
BC =      cm	$m(\hat{A}) = \quad \cdot$
CA =      cm	$m(\hat{B}) = \quad \cdot$

What do you observe?



### Theorem (3)

#### (Angle - Comparison Theorem)

In a triangle, if two sides have unequal lengths, then the longer is opposite to the angle of the greater measure.

**Given:**

ABC in which  $AB > AC$

**R.T.P:**

$m(\hat{ACB}) > m(\hat{ABC})$

**Construction:**

take  $D \in AB$  where  $AD = AC$

**proof:**

in  $\triangle ACD$ ,  $AD = AC$

$\therefore m(\hat{ACD}) = m(\hat{ADC})$  (1)

$\angle ADC$  is an exterior angle of  $\triangle BDC$

$m(\hat{ADC}) > m(\hat{B})$  (2)

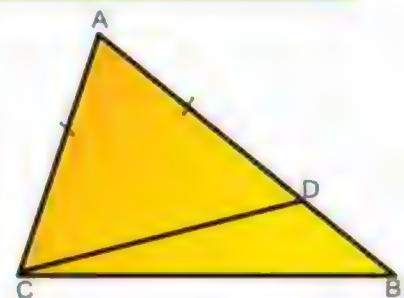
from (1), (2)

$m(\hat{ACD}) > m(\hat{B})$

$m(\hat{ACB}) > m(\hat{ACD})$

$m(\hat{ACB}) > m(\hat{ABC})$

**Q.E.D**

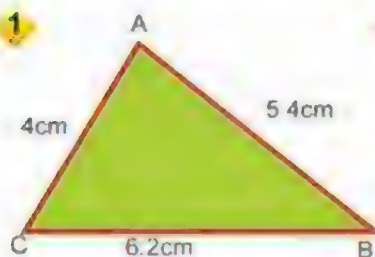




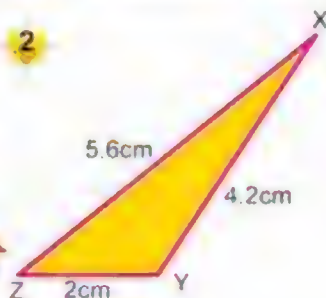
## Practice

In each of the following figures, complete using ( $>$ ,  $<$ )

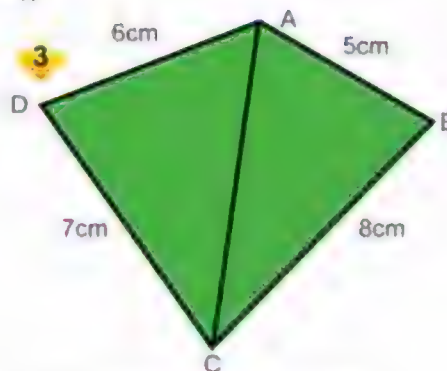
1



2



3



$m(\angle A)$     $m(\angle B)$

$m(\angle A)$     $m(\angle C)$

$m(\angle B)$     $m(\angle C)$

$m(\angle z)$

$m(\angle x)$

$m(\angle z)$

$m(\angle y)$

$m(\angle y)$

$m(\angle x)$

$m(\angle BAC)$     $m(\angle BCA)$

$m(\angle DAC)$     $m(\angle DCA)$

$m(\angle BAD)$     $m(\angle BCD)$

## Remark

The measure of the greatest angle in the triangle  $> 60^\circ$

The measure of the smallest angle in the triangle is  $< 60^\circ$  why?



## Example

In the figure opposite :

ABC is a triangle in which  $AB > BC > CA$

Prove that:  $m(\angle C) > m(\angle A) > m(\angle B)$

**Given:**  $AB > BC > CA$

**R.T.P:**  $m(\angle C) > m(\angle A) > m(\angle B)$

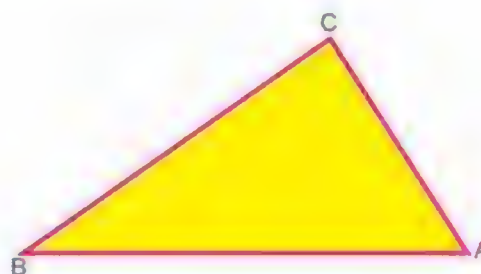
**Proof:** In  $\triangle ABC$

$\therefore AB > BC \quad \therefore m(\angle C) > m(\angle A) \quad (1)$

$\therefore BC > CA \quad \therefore m(\angle A) > m(\angle B) \quad (2)$

from (1), (2) and using the axioms of inequality :

$m(\angle C) > m(\angle A) > m(\angle B)$



## Unit 5 : Lesson 2

### Remember

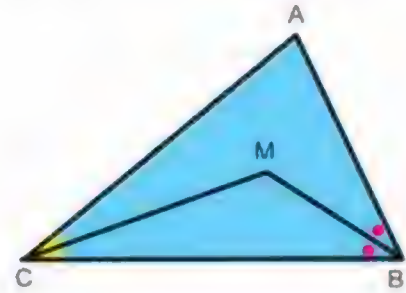
In a triangle, the longest side in length is opposite to the greatest angle in measure while the shortest side in length is opposite to the smallest angle in measure.



### Example

In the figure opposite :

ABC is a triangle where  $\overrightarrow{BM}$  bisects  $\angle ABC$ , and  $\overrightarrow{CM}$  bisects  $\angle ACB$  If:  $MC > MB$



Prove that:  $m(\angle ABC) > m(\angle ACB)$

**Given :**  $\overrightarrow{BM}$  bisects  $\angle ABC$ ,  $\overrightarrow{CM}$  bisects  $\angle ACB$ ,  $MC > MB$ .

**R.T.P:** Prove that  $m(\angle ABC) > m(\angle ACB)$

**Proof:** In  $\triangle MBC$

$$\therefore MC > MB \quad \therefore m(\angle MBC) > m(\angle MCB) \quad (1)$$

In  $\triangle ABC$

$$\therefore \overrightarrow{BM} \text{ bisects } \angle ABC \quad \therefore m(\angle MBC) = \frac{1}{2} m(\angle ABC) \quad (2)$$

$$\therefore \overrightarrow{CM} \text{ bisects } \angle ACB \quad \therefore m(\angle MCB) = \frac{1}{2} m(\angle ACB) \quad (3)$$

$$\therefore \text{from (1), (2), (3)} : \frac{1}{2} m(\angle ABC) > \frac{1}{2} m(\angle ACB) \text{ Using the axioms of inequality}$$

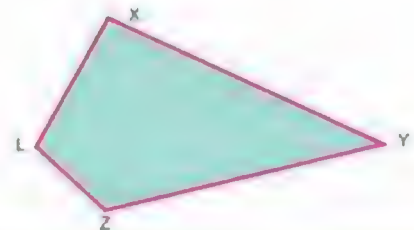
$$\therefore m(\angle ABC) > m(\angle ACB) \quad \text{Q.E.D}$$

## Exercise (5-2)

1.  $\triangle ABC$  in which  $AB = 2.7\text{cm}$ ,  $BC = 8.5\text{cm}$ ,  $AC = 6\text{cm}$ . Order the measures of the angles of the triangle ascendingly.

2. In the figure opposite  $XY > XL$ ,  $YZ > ZL$

Prove that:  $m(\angle XLZ) > m(\angle XYZ)$

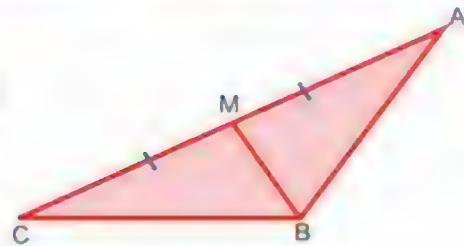




3 In the figure opposite :

$\overline{BM}$  is a median in the triangle  $ABC$ ,  $BM < AM$

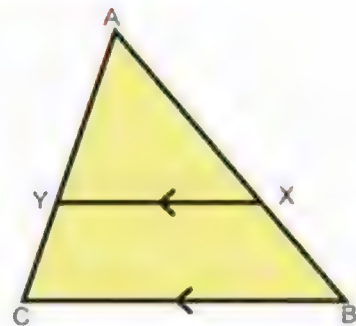
Prove that :  $\angle ABC$  is an obtuse angle.



4 In the figure opposite :

$ABC$  is a triangle,  $AB > AC$ ,  $\overline{XY} \parallel \overline{BC}$

Prove that:  $m(\angle AYX) > m(\angle AXY)$



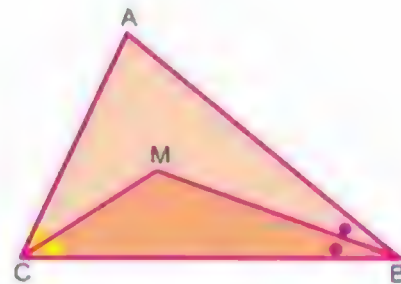
5 In the figure opposite:

$ABC$  is a triangle.  $\overrightarrow{BM}$  bisects

$\angle ABC$ ,

$\overrightarrow{CM}$  bisects  $\angle ACB$ . if  $AB > AC$

Prove that :  $m(\angle MCB) > m(\angle MBC)$



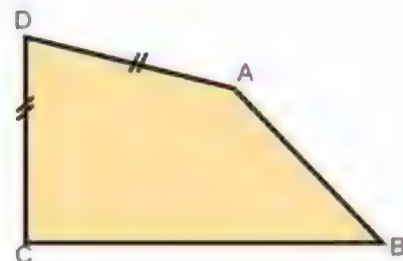
6 In the figure opposite :

$ABCD$  is a quadrilateral in which  $AD = DC$ ,

$BC > AB$

Prove that :

$m(\angle A) > m(\angle C)$



7  $ABCD$  is a quadrilateral in which  $\overline{AB}$  is the longest side in length.  $\overline{CD}$  is the shortest one. Prove that  $m(\angle BCD) > m(\angle BAD)$



# UNIT FIVE

## Lesson Three

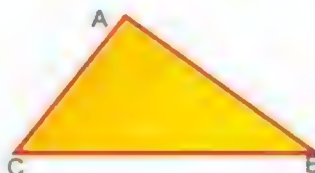
### Comparing the lengths of sides of a triangle

#### Think and Discuss



**Activity 1** The figure opposite: ABC is a triangle of unequal measures of angles.

- ✎ Fold the triangle to make the vertex A coincide vertex B what do you observe regarding to the lengths of the two sides  $\overline{BC}$  and  $\overline{AC}$ , which are opposite to the two unequal angles A and B?
- ✎ Repeat the same previous steps to make vertex B congruent to vertex C. What do you observe?
- ✎ When vertex C is coincide to vertex A, what do you observe?
- ✎ Are there any equal sides in lengths in that triangle?



#### You will learn how

- ✎ To compare the measures of sides in a triangle..

#### key terms

- ✎ The longest side of a triangle.
- ✎ The shortest side of a triangle.
- ✎ the greatest angle of a triangle.
- ✎ the smallest angle of a triangle.
- ✎ The perpendicular line segment.

#### Remark

If the measures of the angles in a triangle are unequal, then the lengths of its sides which are opposite to the angles are unequal.



**Activity 2** Draw the triangle ABC where its angles are unequal in measure then measure the lengths of opposite sides to the angles and complete the following table:

the measures of the angles	the lengths of the opposite sides
$m(\angle A) = \text{ }^\circ$	$BC = \text{ } \text{cm}$
$m(\angle B) = \text{ }^\circ$	$CA = \text{ } \text{cm}$
$m(\angle C) = \text{ }^\circ$	$AB = \text{ } \text{cm}$

**What do you observe?**

- ✎ Is the greatest angle in measure opposite to the longest side in length? Is the smallest angle in measure opposite to the shortest side in length?
- ✎ Is it possible to order the lengths of the sides in the triangle in an ascending or descending order in terms of the measures of the opposite angles?





#### Theorem (4)

#### (Side - Comparison Theorem)

In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

**Given :** In  $\triangle ABC$   $m(\angle C) > m(\angle B)$

**R. T. P :**  $AB > AC$

**Proof :**  $\therefore \overline{AB}, \overline{AC}$  are line segments

$\therefore$  one of the following cases should be verified:

(1)  $AB < AC$

(2)  $AB = AC$

(3)  $AB > AC$

If not  $AB > AC$

Either  $AB = AC$  or  $AB < AC$

if  $AB = AC$ , then  $m(\angle C) = m(\angle B)$

Again this contradicts the given where  $m(\angle C) > m(\angle B)$

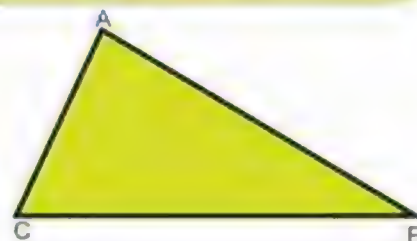
and if  $AB < AC$ , then  $m(\angle C) < m(\angle B)$ . According to the theorem above.

Again this contradicts the given, where

$m(\angle C) > m(\angle B)$

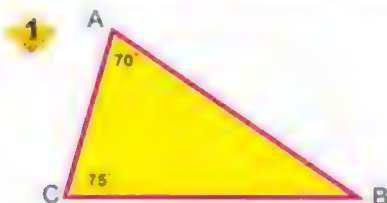
$\therefore AB > AC$

**Q.E.D**



#### practice

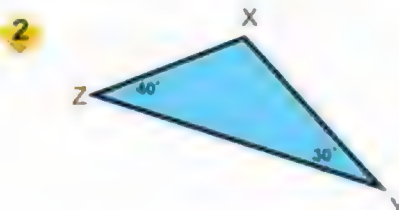
In the following figures, complete using  $>$ ,  $<$  or  $=$ :



$AB$  .....  $AC$

$AB$  .....  $BC$

$AC$  .....  $BC$



$XY$  .....  $XZ$

$YZ$  .....  $XY$

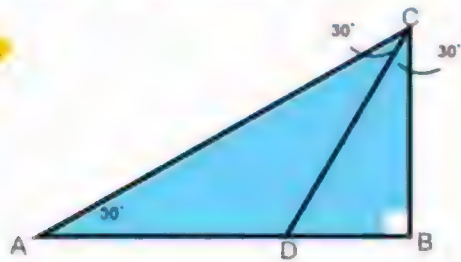
$YZ$  .....  $XZ$





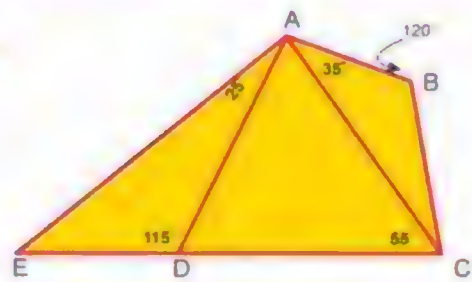
## Unit 5 : Lesson 3

3



$AC \dots\dots BC$   
 $BC \dots\dots DB$   
 $AC \dots\dots BD$   
 $CD \dots\dots AC$

4



$BC \dots\dots AB$   
 $CD \dots\dots CA$   
 $AD \dots\dots AE$   
 $CD \dots\dots AD$

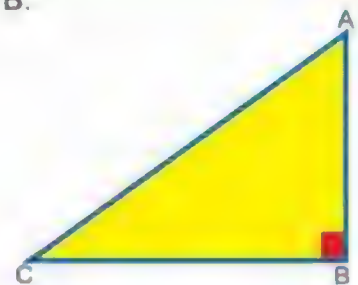
### Corollaries :

#### Corollary (1)

In the right - angled triangle, the hypotenuse is the longest side

In the figure opposite  $\triangle ABC$  is a right - angled triangle at B.

$\therefore \angle A$  acute  $\therefore m(\angle B) > m(\angle A)$   
 $AC > BC$   
 $\therefore \angle C$  acute  $\therefore m(\angle B) > m(\angle C)$   
 $AC > AB$   
 $\therefore AC$  is the longest side Q.E.D.



### Remark :

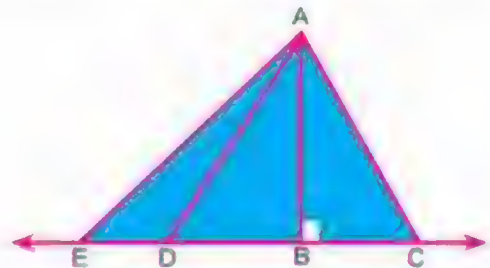
In the obtuse angled triangle, the side opposite to the obtuse angle is the longest side in the triangle .



#### Let's think

$AC > AB$ . Why?  
 $AD > AB$ . Why?  
 $AE > AB$ . Why?

Is the length of the right leg in the right angled-triangle is shorter than the length of the hypotenuse? Why?





### Corollary (2)

The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

### Definition :

The distance between any point and a given straight line is the length of the perpendicular line segment drawn from the point to the given line.



### Example

in the figure opposite:  $ABC$  is a triangle,  $E \in \overrightarrow{BA}$

$\overrightarrow{AD} \parallel \overrightarrow{BC}$ ,  $m(\angle CAD) = 35^\circ$

$m(\angle DAE) = 75^\circ$

Prove that :  $AC > AB$

**Given that:**  $\overrightarrow{AD} \parallel \overrightarrow{BC}$ ,  $m(\angle EAD) = 75^\circ$ ,  $m(\angle DAC) = 35^\circ$

**R.T.P:**  $AC > AB$

**Proof:**  $\because \overrightarrow{AD} \parallel \overrightarrow{BC}$ ,  $\overleftrightarrow{AB}$  is a transversal

$$\therefore m(\angle B) = m(\angle EAD) = 75^\circ$$

Corresponding angles (1)

$\because \overrightarrow{AD} \parallel \overrightarrow{BC}$ ,  $\overleftrightarrow{AC}$  is a transversal

$$\therefore m(\angle ACB) = m(\angle DAC) = 35^\circ$$

alternate angles (2)

From (1) and (2) :

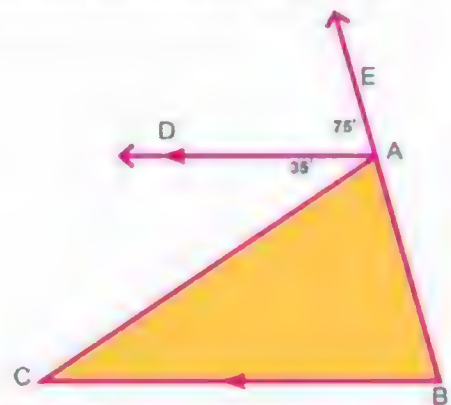
in  $\triangle ABC$

$$m(\angle ABC) = 75^\circ, m(\angle ACB) = 35^\circ$$

$$\text{i.e. } m(\angle ABC) > m(\angle ACB)$$

$$\therefore AC > AB$$

Q.E.D



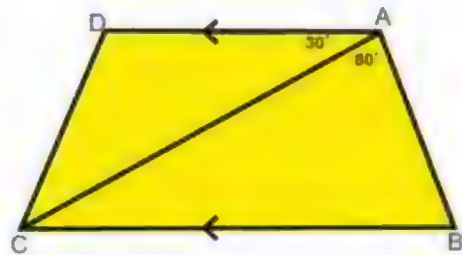
## Exercise (5-3)

- 1  $\triangle ABC$  in which  $m(\angle A) = 40^\circ$ ,  $m(\angle B) = 75^\circ$ , Order the lengths of sides of the triangle descendingly.

- 2 In the figure opposite :

$\overrightarrow{AD} \parallel \overrightarrow{BC}$ ,  $m(\angle BAC) = 80^\circ$

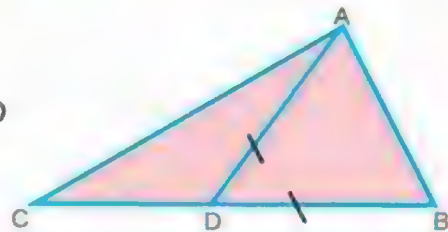
$m(\angle DAC) = 30^\circ$ . Prove that:  $BC > AB$



- 3 In the figure opposite :

In  $\triangle ABC$  is a triangle,  $D \in \overline{BC}$  where  $BD = AD$

prove that :  $BC > AC$

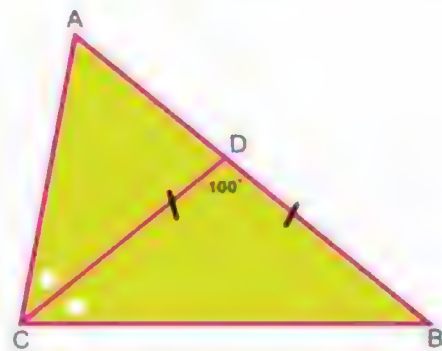


- 4 In the figure opposite :

$ABC$  is a triangle,  $\overrightarrow{CD}$  bisects  $\angle C$  and intersects  $\overline{AB}$  at point  $D$ .

$m(\angle BDC) = 100^\circ$ ,  $DB = DC$

Prove that :  $AC > DB$ .

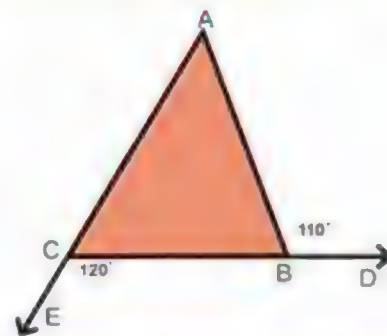


- 5 In the figure opposite :

$ABC$  is a triangle,  $D \in \overrightarrow{CB}$ ,  $E \in \overrightarrow{AC}$

$m(\angle ABD) = 110^\circ$ ,  $m(\angle BCE) = 120^\circ$

Prove that:  $AB > BC$ .

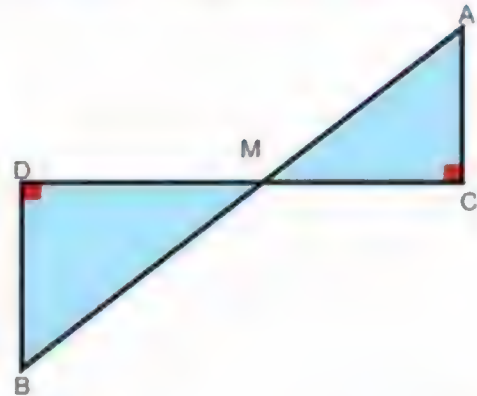




6 In the figure opposite:

$$\overline{AB} \cap \overline{CD} = \{M\}, \overline{AC} \perp \overline{CD}, \overline{BD} \perp \overline{CD}$$

prove that :  $AB > CD$

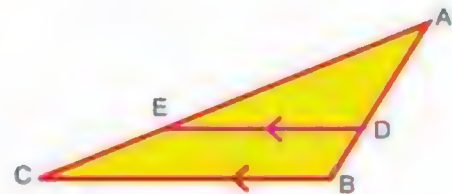


7 In the figure opposite :

$\triangle ABC$  is an obtuse-angled triangle at B

$$\overline{DE} \parallel \overline{BC}$$

Prove that:  $AE > AD$



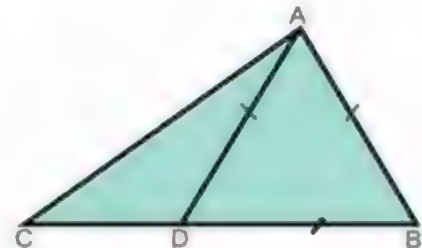
8  $\triangle ABC$  is a triangle,  $\overrightarrow{CD}$  bisects  $\angle C$ ,  $\overrightarrow{CD} \cap \overline{AB} = \{D\}$ , prove that  $BC > BD$

9  $\triangle ABC$  in which  $m(\angle A) = (5x + 2)^\circ$ ,  $m(\angle B) = (6x - 10)^\circ$ ,  $m(\angle C) = (x + 20)^\circ$ , order the lengths of the sides of the triangle ascendingly.

10 In the figure opposite :

$\triangle ABC$  is a triangle,  $D \in \overline{BC}$ ,  $AB = AD = BD$

Prove that :  $BC > AC$



11  $\triangle ABC$  is a right-angled triangle at B,  $D \in \overline{AC}$ ,  $E \in \overline{BC}$ , where  $AD = BE$

Prove that:  $m(\angle CED) > m(\angle CDE)$



# UNIT FIVE

## Lesson Four

### Triangle inequality

#### Think and Discuss



#### Activity

By using your ruler and compass, try to draw the triangle ABC where :

- ①  $AB = 4 \text{ cm}$  ,  $BC = 5 \text{ cm}$  ,  $AC = 6 \text{ cm}$
- ②  $AB = 6 \text{ cm}$  ,  $BC = 3 \text{ cm}$  ,  $AC = 2 \text{ cm}$
- ③  $AB = 9 \text{ cm}$  ,  $BC = 4 \text{ cm}$  ,  $AC = 3 \text{ cm}$
- ④  $AB = 8 \text{ cm}$  ,  $BC = 3 \text{ cm}$  ,  $AC = 5 \text{ cm}$

In which of the previous cases were you able to draw the triangle? What do you conclude?

#### Fact :

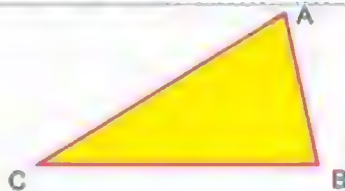
For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

i.e. : In any triangle ABC :

$$AB + BC > AC$$

$$BC + CA > AB$$

$$AB + AC > BC$$



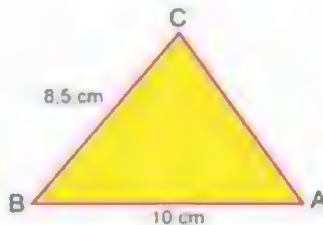
**for example:** the numbers 5, 3 and 9 are not valid to be the lengths of a triangle because the sum of the smallest two numbers  $= 3 + 5 = 8$  ,  $8 < 9$ . Therefore, the inequality of the triangle is not verified.



#### Examples

ABC is a triangle . If  $AB = 10 \text{ cm}$  ,  
 $BC = 8.5 \text{ cm}$

Find the interval which the length of side  $\overline{AC}$  belongs to.



#### You will learn how

- To define the triangle inequality .

#### key terms

- inequality .
- triangle inequality.



### Solution

$$AC < AB + BC$$

$$AC < 18.5 \quad (1)$$

$$\text{However, } AC + BC > AB$$

triangle inequality

$$AC > AB - BC$$

$$AC > 1.5 \quad (2)$$

$$\text{From (1), (2) } 18.5 > AC > 1.5$$

$$AC \in ]1.5, 18.5[$$



### Practice

Find the interval which the third side belongs to in each of the following triangles.

If the lengths of the other two sides were as follows:

- A** 6 cm, 9 cm    **B** 5 cm, 12 cm    **C** 7 cm, 15 cm    **D** 2.9 cm, 3.2 cm

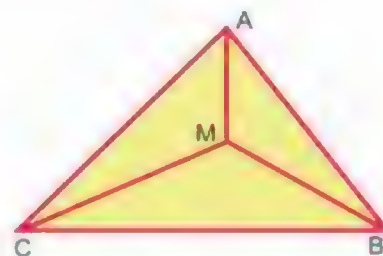
## Exercise (4-4)

- 1 If the lengths of two sides in an isosceles triangle were 5cm, 12cm, Find the length of the third side (State the reason)
- 2 Which of the following groups are valid in drawing a triangle?  
**A** 5 cm, 7 cm, 8 cm    **B** 4 cm, 9 cm, 3 cm  
**C** 10 cm, 6 cm, 4 cm    **D** 15 cm, 17 cm, 30 cm.
- 3 Prove that the length of any side in a triangle is less than half of the perimeter of the same triangle.

- 4 In the figure opposite:

ABC is a triangle in which M is a point inside it. Prove that:

$$MA + MB + MC > \frac{1}{2} \text{ perimeter of the triangle ABC}$$



- 5 prove that : the sum of the lengths of the two diagonals in a convex quadrilateral is less than its perimeter .



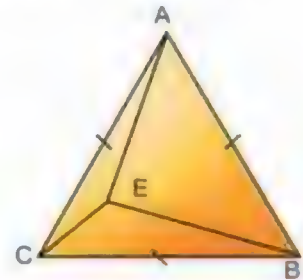


## General Exercises

- 1 In the figure opposite : ABC is an equilateral triangle where E is a point inside it  
 $m(\angle ECB) > m(\angle EBC)$ .

First: prove that :  $m(\angle ABE) > m(\angle ACE)$ .

Second:  $m(\angle A) > m(\angle ABE) > m(\angle ACE)$ .

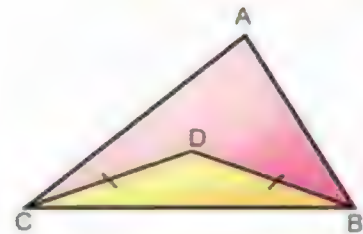


- 2 In the figure opposite :

$$DB = DC.$$

$$m(\angle ABC) > m(\angle ACB)$$

Prove that:  $m(\angle ABD) > m(\angle ACD)$

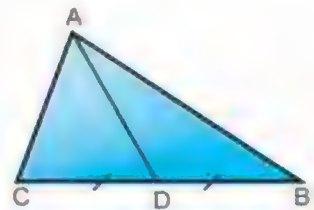


- 3 ABC is a triangle in which  $AB = 6\text{cm}$ ,  $AC = 7\text{cm}$ ,  $BC = 8\text{cm}$ .  
 Order the measures of its angles ascendingly.

- 4 In the figure opposite :

$$AB > AC, DB = DC$$

Prove that:  $m(\angle BAD) < m(\angle CAD)$ .

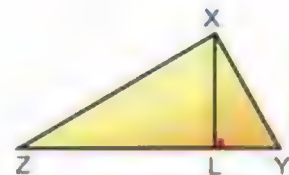


- 5 In the figure opposite :

$$XZ > XY$$

$$XL \perp ZY$$

prove that  $m(\angle LXZ) > m(\angle LXY)$



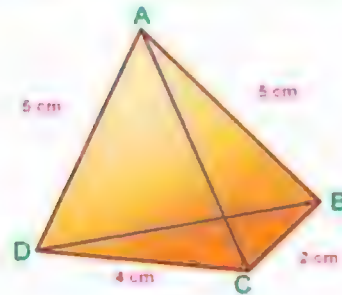
6 In the figure opposite :

ABCD is a quadrilateral in which

$AB = AD = 5\text{ cm}$ ,

$BC = 2\text{ cm}$ ,  $DC = 4\text{ cm}$ .

Prove that:  $m(\hat{ABC}) > m(\hat{ADC})$

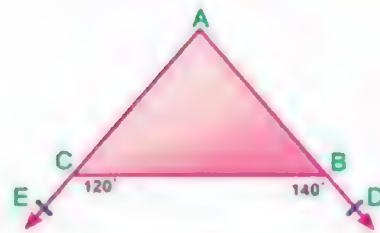


7 In the figure opposite :

$m(\hat{DBC}) = 140^\circ$

$m(\hat{ECB}) = 120^\circ$

Prove that:  $CB > AB$



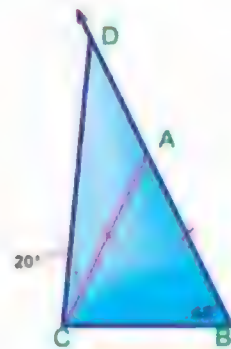
8 In the figure opposite :

$AB = AC$

$m(\hat{ABC}) = 65^\circ$

$m(\hat{ACD}) = 20^\circ$

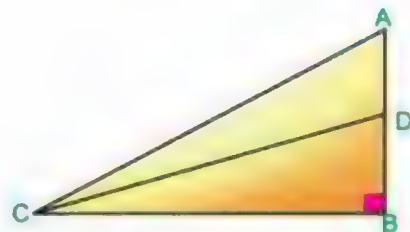
Prove that:  $AB > AD$



9 In the figure opposite :

$m(\hat{B}) = 90^\circ$

Prove that:  $AC > DC$



10 In the figure opposite :

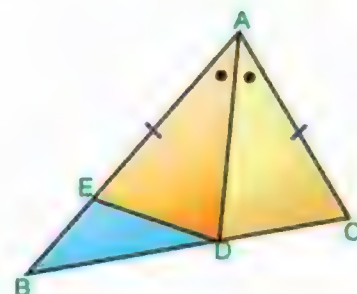
$AC > DC$ ,  $m(\hat{CAD}) = m(\hat{BAD})$

$AE = AC$

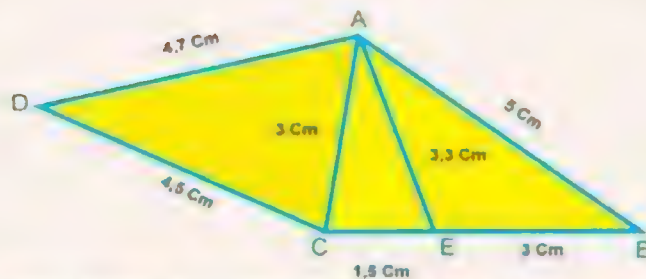
Prove that: A  $DE = DC$

B  $m(\hat{BAD}) > m(\hat{ADC})$

C  $BD > DC$ .



## Activity



1 In the given figure complete using ( $<$  or  $>$ )

- A  $m(\angle DAC) \dots\dots\dots m(\angle ACD)$
- B  $m(\angle AEC) \dots\dots\dots m(\angle ECA)$
- C  $m(\angle ABE) \dots\dots\dots m(\angle EAB)$
- D  $m(\angle CDA) \dots\dots\dots m(\angle DAC)$
- E  $m(\angle AEB) \dots\dots\dots m(\angle EAC)$

2 In the triangle ABC,  $AB = 6\text{ cm}$ ,  $BC = 9\text{ cm}$

then  $AC \in ] \dots\dots\dots , \dots\dots\dots [$

3 In the triangle ABC :  $m(\angle A) = (9x)^\circ$ ,  $m(\angle B) = (6x - 17)^\circ$   
 $m(\angle C) = (7x - 1)^\circ$

Order the lengths of the sides of the triangle ascendingly.





# Unit test

1 Complete the following to make each statement true:

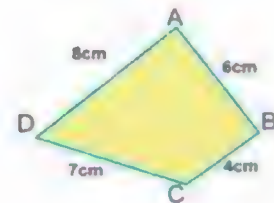
- A In a triangle, the smallest angle in measure is opposite to .....
- B In  $\triangle ABC$ : if  $m(\angle A) = 70^\circ$ ,  $m(\angle B) = 30^\circ$ , then the longest side in length is .....
- C If the lengths of two sides in an isosceles triangle were : 3 cm, 7cm then ,  
the length of the third side = ..... cm
- D  $\triangle ABC$  in which:  $m(\angle A) = 100^\circ$ , then the greatest side length is .....
- E  $\triangle ABC$  in which  $AB = 3\text{cm}$ ,  $BC = 5\text{cm}$ , then  $AC \in ]....., .....[$
- F The longest side length in the right-angled triangle is .....

2 In the figure opposite :

ABCD is a quadrilateral in which  
 $AB = 6\text{cm}$ ,  $BC = 4\text{cm}$ ,  
 $CD = 7\text{cm}$ ,  $DA = 8\text{cm}$

Prove that:

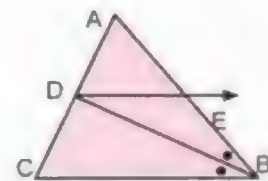
$$m(\angle BCD) > m(\angle BAD)$$



3 In the figure opposite :

$\triangle ABC$  is a triangle,  $\overrightarrow{BD}$  bisects  $\angle B$ ,  $\overrightarrow{BD} \cap \overrightarrow{AC} = \{D\}$ ,  
 $\overrightarrow{DE} \parallel \overrightarrow{CB}$  and intersects  $\overrightarrow{AB}$  at E

Prove that:  $AB > AD$

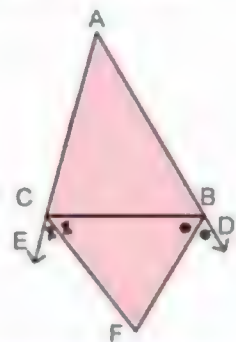


4 In the figure opposite :

$\triangle ABC$  in which  $AB > AC$ ,  $D \in \overrightarrow{AB}$ ,  $E \in \overrightarrow{AC}$   
 $\overrightarrow{BF}$  bisects  $\angle DBC$ ,  $\overrightarrow{CF}$  bisects  $\angle BCE$   
 $\overrightarrow{BF} \cap \overrightarrow{CF} = \{F\}$

Prove that :

- A  $m(\angle FBC) > m(\angle BCF)$
- B  $CF > BF$



## Model tests on Algebra and statistics (2nd preparatory)

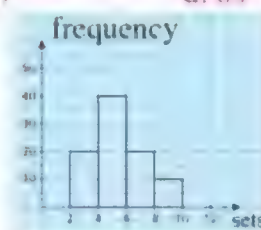
### Model (1)

**First question :** complete the following :

- (1) The S.S of the equation  $(x^2 + 3)(x^3 + 1) = 0$  is ..... :  $x \in \mathbb{R}$
- (2) If the lowest boundary of a set is 10 and the upper boundary is  $x$  and its centre is 15, then  $x =$  .....
- (3)  $]-2, 2] \cup \{-2, 0\} =$  .....
- (4) A cube whose volume is  $8 \text{ cm}^3$ , then the sum of the lengths of all its edges equals ..... cm.
- (5) The multiplicative inverse of the number  $\sqrt{3} + \sqrt{2}$  is ..... (in the simplest form).

**Second question :** Choose the correct answer from those given.

- (1) If the radius of a sphere is 6 cm , then its volume is .....  
 a.  $6 \pi \text{ cm}^3$       b.  $36 \pi \text{ cm}^3$       c.  $72 \pi \text{ cm}^3$       d.  $288 \pi \text{ cm}^3$
- (2) If the point (a,1) satisfies the relation  $x+y=5$ , then  $a=$  .....  
 a. 1      b. -4      c. 4      d. 5
- (3)  $(2\sqrt[3]{2})^3 =$  .....  
 a. 4      b. 8      c. 16      d. 40
- (4) The median of the values 34 , 23 , 25 , 40 , 22 , 4 is .....  
 a. 22      b. 23      c. 24      d. 25
- (5) If the arithmetic mean of the values 27 , 8 , 16 , 24 , 6 , k is 14 then  $k =$  .....  
 a. 3      b. 6      c. 27      d. 84
- (6) In the opposite figure  
 the value of the mode = .....  
 a. 4      b. 5  
 c. 6      d. 40



**question (3)**

- (a) Find the value of  $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$
- (b) If  $x = \frac{3}{\sqrt{3}-\sqrt{2}}$  ,  $y = \sqrt{5} - \sqrt{2}$  , prove that:  $x, y$  are two conjugate

**question (4)**

- (a) The area of a square is  $1089 \text{ cm}^2$ . Find the length of its diagonal.
- (b) Find the S.S of the inequality  $\frac{3x+1}{6} < x+1 < \frac{x+4}{2}$  in  $\mathbb{R}$  , then represent it on the number line.



**question (5)**

- (a) The radius length of the base of a right circular cylinder , its base radius length is  $4\sqrt{2}$  cm and its height is 9 cm find its volume in terms of  $\pi$  and if its volume equals the volume of a sphere find the radius length of the sphere.
- (b) Find the arithmetic mean of the following frequency distribution.

The set	5 –	15 –	25 –	35 –	45 –	total
Frequency	7	10	12	13	8	50

**Model (2)**

**Question one : Complete the following :**

- (1) The additive inverse of the number  $-\sqrt{3} - \sqrt{5}$  is .....
- (2)  $(\sqrt{8} + \sqrt{2})(\sqrt{8} - \sqrt{2}) = \dots\dots\dots$
- (3) The conjugate of the number  $\frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$  is .....
- (4) If the volume a sphere is  $\frac{9}{2}\pi \text{ cm}^3$  then its diameter length is ..... cm.
- (5)  $\{3,4\} - \{3,5\} = \dots\dots\dots$

**Question (2) : Choose the correct answer from those given :**

- (1) If the volume of a cube is  $27 \text{ cm}^3$  then the area of one of its faces is .....  
 a.  $3 \text{ cm}^2$                       b.  $9 \text{ cm}^2$                       c.  $36 \text{ cm}^2$                       d.  $54 \text{ cm}^2$
- (2) If the mode of the set of values 4 , 11 , 8 ,  $2\chi$  is 4 then  $\chi = \dots\dots\dots$   
 a. 2                                  b. 4                                  c. 6                                  d. 8
- (3) If the arithmetic mean of the set of values 18 , 23 , 29 ,  $2k-1$  , k is 18 then k = .....  
 a. 1                                  b. 7                                  c. 29                                  d. 90
- (4) If the lowest limit of a set is 4 and the upper limit is 8 then its centre is .....  
 a. 2                                  b. 4                                  c. 6                                  d. 8
- (5) A right circular cylinder, the radius of its base is r, its height equals the length of the diameter, then its volume= .....  
 a.  $\pi r^3$                               b.  $\pi r^2$                               c.  $2\pi r^3$                               d.  $2r^3$
- (6) The sloution set in R of the equation  $X(X^2 - 1) = 0$  is .....  
 a.  $\{0\}$                               b.  $\{1\}$                               c.  $\{-1\}$                               d.  $\{0,-1,1\}$





**question (3)**

(a) Reduce to the simplest form.  $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

(b) Prove that  $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

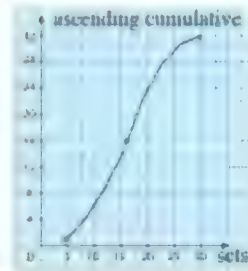
**question (4)**

(a) Find the S.S of the inequality  $-2 < 3x + 7 \leq 10$  in  $\mathbb{R}$ , then represent the interval of solution on the number line.

(b) If  $x = \sqrt{2} + \sqrt{3}$  find the value of  $x^4 - 2x^2 + 1$

**question (5)**

(a) The opposite figure represents the ascending cumulative curve of the marks 32 students in one test complete the median mark = .....



(b) Find the arithmetic mean of the following frequency distribution.

The set	5 -	15 -	25 -	35 -	45 -	total
Frequency	4	5	6	3	2	20

### Model Test (3)

#### "Merge students"

**Questions (1)** Complete each of the following:

- 1) The conjugate of the number  $\sqrt{3} + \sqrt{2}$  is .....
- 2)  $\sqrt{18} + \sqrt{54} - 3\sqrt{2} = \dots\dots\dots$
- 3) The mode for the number 3 , 5 , 3 , 4 , 3 is .....
- 4) The median of the values 2 , 3 , 5 , 7 , 9 is .....
- 5) The solution set of the equation  $x^2 + 9 = 0$  in  $\mathbb{R}$  is .....

**Questions (2)** Choose correct answer:

- 1) The arithmetic mean for the values 9 , 6 , 5 , 14 , 1 is .....  
 (a) 7 (b) 3 (c) 5 (d) 9
- 2) The simplest form of the expression  $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$  is .....  
 (a)  $\sqrt{3}$  (b) 1 (c)  $\sqrt{2}$  (d)  $2\sqrt{3}$
- 3) The additive inverse of the number  $-\sqrt{5}$  is .....  
 (a)  $\sqrt{5}$  (b) 5 (c)  $\sqrt{2}$  (d) -5




4)  $\{3, 5\} - \{3, 5\} = \dots\dots\dots$

- (a)  $\{3, 5\}$  (b)  $[3, 5]$  (c)  $\phi$  (d)  $]3, 5[$

5) A cube of volume  $64 \text{ cm}^3$ , then the edge length is  $\dots\dots\dots$  cm.

- (a) 4 (b) 8 (c) 16 (d) 64

**Questions (3)** Match from the column (A) to the suitable one from the column (B):

(A)	(B)
1) The solution set of the equation $x^2 - 25 = 0$ is	$[0, 2]$
2) $[-3, 2] \cap [0, 2] = \dots\dots\dots$	7
3) If the order of the median is the fourth, then the number of the values is $\dots\dots\dots$	$\{5, -5\}$
4) $\sqrt{3}$ is an $\dots\dots\dots$ number.	
5) The solution set of the inequality $3 \leq x \leq 7$ on the number line is $\dots\dots\dots$	Irrational

**Questions (4)** Put (✓) for the correct statement, (x) for the incorrect one:

- The arithmetic mean for the a set of values = the sum of these values + its number. ( )
- If  $x = \sqrt{13} - \sqrt{7}$ ,  $y = \sqrt{13} + \sqrt{7}$ , then  $x, y$  are two conjugate numbers. ( )
- The irrational number  $\sqrt{7}$  lies between 2 and 3 ( )
- $\sqrt{75} - 2\sqrt{27} = 7\sqrt{3}$  ( )
- The simplest from of the number  $\frac{1}{\sqrt{5}}$  is  $\frac{\sqrt{5}}{5}$  ( )

**Questions (5)** First: Complete:

If the lower limit of a set is 4, the upper limit is 8, then its centre =  $\frac{\dots\dots + \dots\dots}{2} = \dots\dots\dots$



## Second : Complete

The following table to obtain the arithmetic mean of the following frequency distribution:

Sets	5-	15-	25-	35-	45-	Total
Frequency	7	10	12	13	8	50

Sets	The centre of the set x	Frequency (F)	F x X
5 -	10	7	$10 \times 7 = 70$
15 -	20	10	$20 \times 10 = \dots\dots\dots$
25 -	$\dots\dots\dots$	$\dots\dots\dots$	$\dots\dots \times 12 = \dots\dots\dots$
35 -	$\dots\dots\dots$	$\dots\dots\dots$	$\dots\dots \times 13 = \dots\dots\dots$
45 -	$\dots\dots\dots$	$\dots\dots\dots$	$\dots\dots \times 8 = \dots\dots\dots$
The sum	$\dots\dots\dots$	50	$\dots\dots\dots$

The arithmetic mean =  $\frac{\Sigma(F \times X)}{\Sigma(F)} = \frac{\dots\dots\dots}{\dots\dots\dots} = \dots\dots\dots$





## Model tests on Geometry 2 nd preparatory

### Model (1)

#### 1 Complete the following :

The longest side in the right angled triangle is .....

If the lengths of two sides in a triangle are 2 cm. and 7 cm. then .....  $\angle$  the length of the third side < .....

If the measures of two angles in a triangle are different, then the greater in measure of them is opposite to .....

If the median drawn from a vertex of a triangle equals half the opposite side to this vertex in length then .....

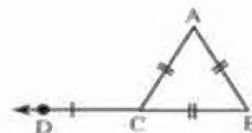
If the measure of an angle in the isosceles triangle equals  $60^\circ$  then the triangle is .....

#### 2 Choose the correct answer from those given :

In the opposite figure :

$\Delta ABC$  is equilateral , then  $m(\angle ACD) = \dots\dots\dots$

- (a)  $45^\circ$  (b)  $60^\circ$   
(c)  $120^\circ$  (d)  $135^\circ$



In  $\Delta ABC$  which is right angled at B , if  $AC = 20$  cm. then the length of the median of the triangle drawn from B equals .....

- (a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.

$XYZ$  is a triangle in which  $m(\angle Z) = 70^\circ$  and  $m(\angle Y) = 60^\circ$  then  $YZ \dots\dots\dots XY$

- (a) > (b) < (c) = (d) twice

The lengths which can be lengths of sides of a triangle are

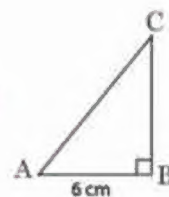
- (a) (0 , 3 , 5) (b) (3 , 3 , 5) (c) (3 , 3 , 6) (d) (3 , 3 , 7)

The triangle in which the measure of two angles of it are  $42^\circ$  and  $69^\circ$  is .....

- (a) an isosceles triangle. (b) an equilateral triangle.  
(c) a scalene triangle. (d) a right angled triangle.

In the opposite figure,  $m(\angle A) = 2m(\angle C)$ ,  $AC = \dots\dots\dots$  cm.

- (a) 3 (b) 6 (c) 9 (d) 12



#### 3 Complete :

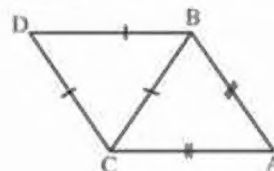
[a] If  $\Delta ABC$  in which  $AB > AC$  then  $m(\angle C) \dots\dots\dots m(\angle B)$

[b] In the opposite figure :

$m(\angle A) = 50^\circ$ ,  $AB = AC$

and  $\Delta DBC$  is an triangle equilateral

Find  $m(\angle ABD)$

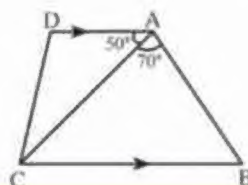


[c] In the opposite figure :

$AD \parallel BC$ ,  $m(\angle BAC) = 70^\circ$

and  $m(\angle DAC) = 50^\circ$

Prove that :  $BC > AC$

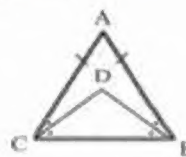


- 4 [a] Prove that the base angles of the isosceles triangle are congruent.

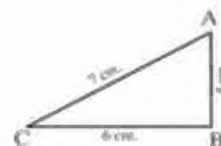
[b] In the opposite figure :

$AB = AC$ ,  $\overrightarrow{BD}$  bisects  $\angle B$  and  $\overrightarrow{CD}$  bisects  $\angle C$

Prove that :  $\triangle DBC$  is isosceles



- 5 [a] In the opposite figure :  
Arrange the angles  
of  $\triangle ABC$  descendingly  
due to their measures

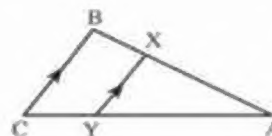


[b] In the opposite figure :

$AB > BC$ ,  $XY \parallel BC$

Prove that :

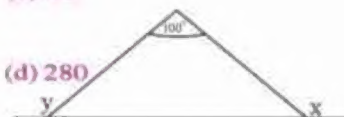
$AX > XY$



## Model (2)

- 1 Choose the correct answer from those given :

- The triangle which has three axes of symmetry is .....  
(a) scalene (b) isosceles (c) right angled (d) equilateral.
- The sum of lengths of two sides in a triangle is ..... the length of the third side.  
(a) greater than (b) smaller than (c) equals to (d) twice
- If the lengths of two sides in an isosceles triangle are 8 cm. and 4 cm. then the length of the third side is .....  
(a) 4 (b) 8 (c) 3 (d) 12
- In  $\triangle ABC$  if  $m(\angle B) = 130^\circ$  then the tallest side of it is .....  
(a)  $\overline{BC}$  (b)  $\overline{AC}$  (c)  $\overline{AB}$  (d) its median
- $\triangle XYZ$  is an isosceles triangle in which  $m(\angle X) = 100^\circ$  then  $m(\angle Y) = \dots^\circ$   
(a) 100 (b) 80 (c) 60 (d) 40
- In the opposite figure  $x+y = \dots^\circ$   
(a) 100 (b) 140 (c) 180 (d) 280



- 2 Complete the following :

- If the measure of an angle in a right angled triangle is  $45^\circ$  then the triangle is .....
- The length of any side in a triangle ..... the sum of lengths of the two other sides.
- If  $\overline{AB} \cong \overline{XY}$  then  $AB = \dots$
- In  $\triangle ABC$  of  $m(\angle A) = 30^\circ$  and  $m(\angle B) = 90^\circ$  then  $BC = \dots AC$
- The axis of symmetry of a line segment is the straight line which ..... at its midpoint.

- 3 [a] In  $\triangle ABC$  in which  $AB = 7$  cm. ,  $BC = 5$  cm. and  $AC = 6$  cm. Arrange its angles ascendingly due to their measure,

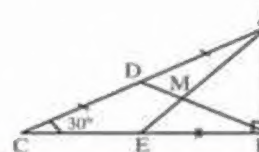
[b] In the opposite figure :

$\triangle ABC$  is right angled at B ,

$m(\angle C) = 30^\circ$ , D is the midpoint of  $\overline{AC}$

, E is the midpoint of  $\overline{BC}$ ,  $AC = 9$  cm.

Find the length of each of  $\overline{BD}$ ,  $\overline{BM}$  and  $\overline{AB}$

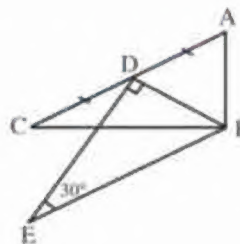




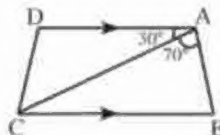
- 4 [a] In the opposite figure :  
 $m(\angle ABC) = m(\angle BDE) = 90^\circ$   
 $m(\angle E) = 30^\circ$

D is the midpoint of  $\overline{AC}$

Prove that :  $AC = BE$

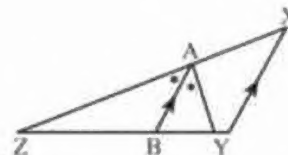


- [b] In the opposite figure :  
 $\overline{AD} \parallel \overline{BC}$ ,  $m(\angle BAC) = 70^\circ$   
 $m(\angle DAC) = 30^\circ$   
 Prove that :  
 $AC > BC$



- 5 [a] If the measures of two angles of a triangle are different, then their greater in measure is opposite to .....

- [b] In the opposite figure :  
 $\overline{AB} \parallel \overline{XY}$  and  $\overline{AB}$  bisects  $\angle YAZ$   
 Prove that :  
 $XZ > YZ$



## Model Test

### " Merge students "

#### Questions (1) Complete:

- The point of concurrence of the medians of the triangles divides each median in the ratio of ..... : ..... from its .....
- In the right-angled triangle, the length of the median drawn from the right angle equals .....
- The base angles of the isosceles triangle are .....
- In  $\triangle ABC$ ,  $m(\angle B) = 70^\circ$ ,  $m(\angle C) = 50^\circ$ , then  $AC$  .....  $AB$
- The median of an isosceles triangle from the vertex angle .....

#### Questions (2): Choose the correct answer:

- If  $\triangle ABC$  is an equilateral triangle, then  $m(\angle B) = \dots\dots\dots^\circ$   
 (a)  $30^\circ$  (b)  $60^\circ$  (c)  $70^\circ$  (d)  $90^\circ$
- The length of the side opposite to angle of  $30^\circ$  in the right-angled triangle = ..... Hypotenuse.  
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{4}$  (d) 2





3) If the measure of the vertex angle of the isosceles triangle equals  $80^\circ$ , then the measure of one of the base angles = .....  $^\circ$ .

- (a) 60                      (b) 40                      (c) 30                      (d) 50

4) The number of axes of symmetry of the isosceles triangle = .....

- (a) 1                      (b) 2                      (c) 3                      (d) 0

5)  $\triangle ABC$ ,  $m(\angle A) = 50^\circ$ ,  $m(\angle B) = 60^\circ$ , then the longest side is .....

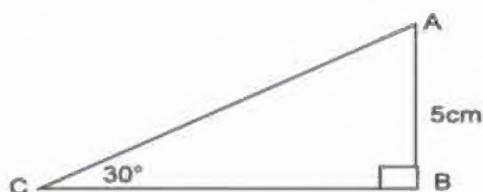
- (a)  $\overline{AB}$                       (b)  $\overline{BC}$                       (c)  $\overline{AC}$

**Questions (3):**      In the opposite figure:

ABC is right-angled triangle at B,

$m(\angle C) = 30^\circ$ ,  $AB = 5$  cm,

Find the length of  $\overline{AC}$



$\therefore m(\angle B) = \dots\dots\dots^\circ$ ,  $m(\angle C) = \dots\dots\dots^\circ$

$\therefore AB = \frac{1}{2} \times \dots\dots\dots$

$\therefore AC = \dots\dots\dots$  cm

**Questions (4):**

A)  $\triangle ABC$ ,  $m(\angle A) = 40^\circ$ ,  $m(\angle B) = 75^\circ$ ,  $m(\angle C) = 65^\circ$ , arrange the lengths of the sides of this triangle descendingly the order is : .....

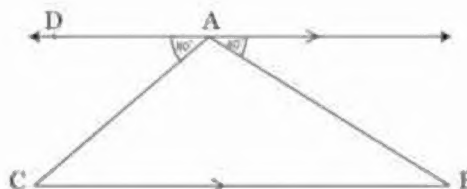
B) In the opposite figure:

$\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

Complete:

1-  $m(\angle B) = \dots\dots\dots^\circ$

2- The longest side of  $\triangle ABC$  is .....



**Questions (5):**      In the opposite figure Put ( $\checkmark$ ) in front of the correct answer, ( $\times$ ) for the incorrect answer:

$AB = AC = CD = AD = 10$  cm ,  $m(\angle BAC) = 70^\circ$

- |                                |          |
|--------------------------------|----------|
| 1) $m(\angle B) = 55^\circ$    | (      ) |
| 2) $m(\angle D) = 70^\circ$    | (      ) |
| 3) $m(\angle DCB) = 120^\circ$ | (      ) |
| 4) $AB + AD = 20$ cm           | (      ) |
| 5) $AB + BC = BC + CD$         | (      ) |

